STATIC AND KINEMATIC BEHAVIOUR OF A FOLDABLE TRUSS ROOF STRUCTURE

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ABSTRACT: In this paper, a type of foldable truss structure formed by four-bar linkages is firstly proposed as a retractable roof. Then the mechanical behavior of a fully closed roof system is studied. The results show that the member buckling has a great effect on the ultimate capacity of foldable truss structures. The failure load of the structure is smaller if buckling is considered. It is also found that the effect of imperfections on the failure load of the foldable structure is not significant. Furthermore, the stress variation during the folding is small. However, when initial imperfections are introduced, the maximal member stress during the motion increases greatly, which is almost close to yield stress.

Keywords: Foldable structure, four-bar linkage, moving process, mechanical behaviour, cables, imperfection

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1. INTRODUCTION

Pantographic foldable structure, which uses scissor-like elements (SLEs) as basic structural elements, has many potential earthbound and aerospace industry applications [1]. In 1960s, the first of such structure has been designed and constructed by Spanish architect Piñero [2]. Substantial contributions to the general understanding of geometric and kinematic behavior of SLEs are due to Escrig [3, 4] and Escrig and Valcarcel [5]. They also applied this concept to the design of a swimming pool [6].

Zeigler firstly investigated the ‘snap-through’ phenomenon during the motion, which is caused by geometric incompatibilities between the SLEs [7]. Then the geometry design and structural response during the deployment of snap-through structures were thoroughly investigated by Gantes *et al*. [8], Gantes [9], Gantes and Konitopoulou [10]. Further studies of such deployable structures based on SLEs have been made by many other researchers. Shan presented an approach based on the concept of the standard stiffness method for the computer analysis of foldable structures [11]. Kaveh and Davaranl also developed an efficient computational method, incorporating the stiffness matrix of a SLE into a standard stiffness method [12]. Langbecker [13] formulated the foldability equation using a purely geometric approach, which then was used to analyze the kinematics and determine the foldability/deployability of translational, cylindrical, and spherical configurations. The kinematic of pantograph structures has also been studied by obtaining the null space of a constraint Jacobian matrix or the Screw theory [14, 15]. A scissor-hinged system consisting of several interconnected parallel linear SLEs was suggested as the supporting structure for membrane structures [16].

The elements of SLEs on the above study are straight bars. In the early 1990's, Hoberman invented a method for constructing loop assemblies consisting of a pair of identical angulated rods connected by a scissor hinge [17, 18]. In analogy to SLEs made from straight rods, angulated SLEs subtend a constant angle as their rods rotate while maintaining the end pivots on parallel lines. You and Pellegrino used the previous concept onto multi-angulated SLEs having more than one kink angle [19]. The cover elements and support conditions of the retractable roof structure based on angulated SLEs were also investigated by Kassabian *et al*. [20], Buhl and Jensen [21], and Jensen [22]. Furthermore, kinematics of Hoberman's Linkages was also studied using the Screw theory [23, 24]. Several other types of closed loop structures have been developed and studied [25-30].

Retractable roofs are structures that can transform from one configuration to another in order to provide a variable cover to the space underneath in response to changing conditions and/or functional requirements [31]. From a structural point of view, retractable roof structures have to be designed for two completely different loading conditions in the open and close situations. The structural design process is very complicated and requires successive iterations to achieve some balance between desired flexibility during deployment and higher stiffness in the open configuration [1, 10]. However when the foldable truss based on SLEs was used as a retractable roof structure, Chen *et al*. [32] suggested that there are still many unsolved problems such as high bending moment of member and low structural stiffness etc. The large bending moment will reduce the structural efficiency and low structural stiffness will lead to larger deflection. Moreover, Teall also observed that the deformation of the structure was quite large under self-weight in the experimental investigation of the structural behavior of a three layer foldable dome based on multi-angulated SLEs with a span of 2 m [33]. This leads to low stiffness of this structure that is unable to resist external loads. Recently, Mao and Luo investigated a few support conditions and structural forms to increase the structural stiffness [30].

In this paper, a foldable truss roof structure based on the four-bar linkage is introduced. An active cable and a passive cable are added between the diagonal joints of the four-bar linkage to control the deployment process and increase the structural stiffness. The mechanical behavior of the proposed system in the fully deployed configuration was then investigated and the role of cables was studied. Moreover, the moving process of the system was simulated using commercial software ABAQUS. The influence of the imperfection on the behavior in motion was also discussed.

2. GEOMETRICAL DESIGN

Plane linkages are member assemblies using revolute pairs or prismatic pairs. The simplest example of them is the planar four-bar linkage, which is widely used and works as the foundation of other planar linkages. The four-bar linkages in this paper is connected by revolute hinges only with a rotational degree-of-freedom perpendicular to the mechanism plane.

The foldable unit based on four-bar linkages, shown in Figure 1, is driven by telescopic rods, in which the four bars on the edge is neither foldable nor contractile while the diagonal bar can change its length as designed as a telescopic rod. Then at the foldable configuration, the structure is bunch-like and corresponding elements are parallel to each other. With the elongation of the telescopic rod, the structure is gradually unfolded and reaches its fully deployed state with the telescopic rods fully extended. In order to deploy the structures fully, the length of bars, l1 to l4, should satisfy the following equation as

$$l_1 + l_3 = l_2 + l_4 \tag{1}$$

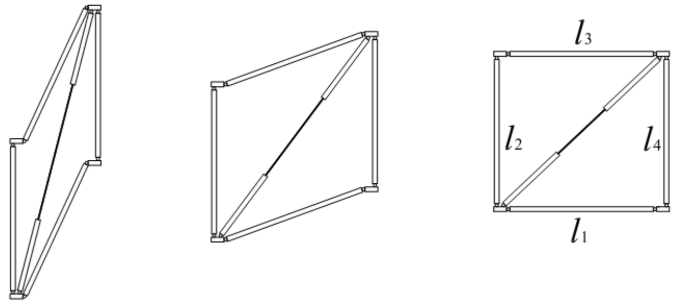


Figure 1. Foldable Units Driven by the Telescopic Rod

When this foldable unit is used to develop a deployable structure, large number of telescopic rods are required to drive the system, which makes it practically difficult for large deployable structures. Another method, based on the idea of active cables and passive cables [34], is proposed in this paper. In the foldable configuration, both the active cable and passive cable are slack as shown in Figure 2(a). During the deployment of the system, the distance between nodes A and C decreases progressively along with the shortening of the active cable. During the motion, the active cable is always in tension while the passive cable is slack. Subsequently, with the further shortening of the active cable, the passive cable turns to be tensional and thus the system movement is terminated as shown in Figure 2(c). If we keep stretching the active cable, no rigid-body displacement is produced but the whole system has been imposed with prestress shown in Figure 2(d).

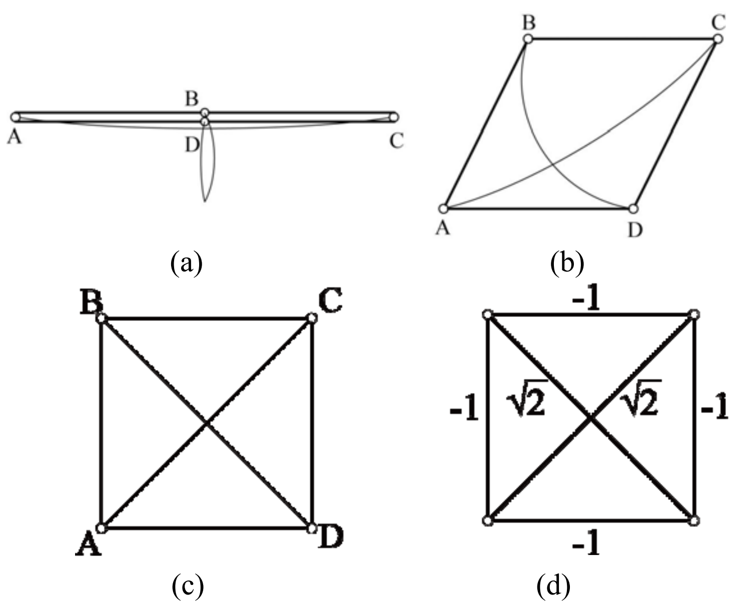


Figure 2. Foldable Units Driven by Cables

Due to its advantages of light weight, large rigidity, high integrity and well seismic performance, the space grid structure is widely used in various long-span structures. A spatial foldable unit, which is developed by the planar foldable unit based on four-bar linkages shown in Figure 2, will be used to form a foldable space truss structure in this paper. The moving progress of the spatial foldable unit is shown in Figure 3. If we consider each plane in this unit as a bar, then the unit can be considered as a four-bar linkage.

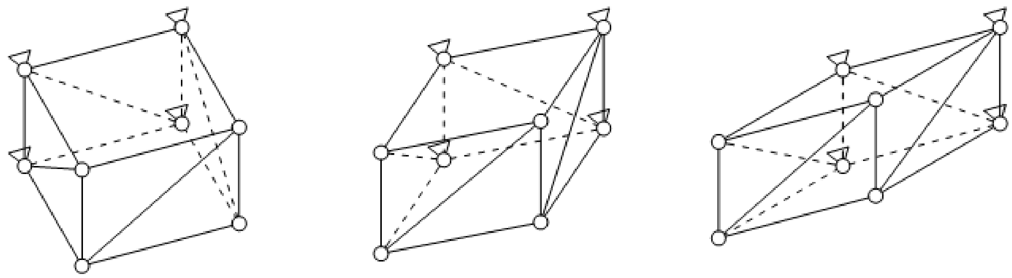


Figure 3. Moving Progress of the Spatial Foldable Unit

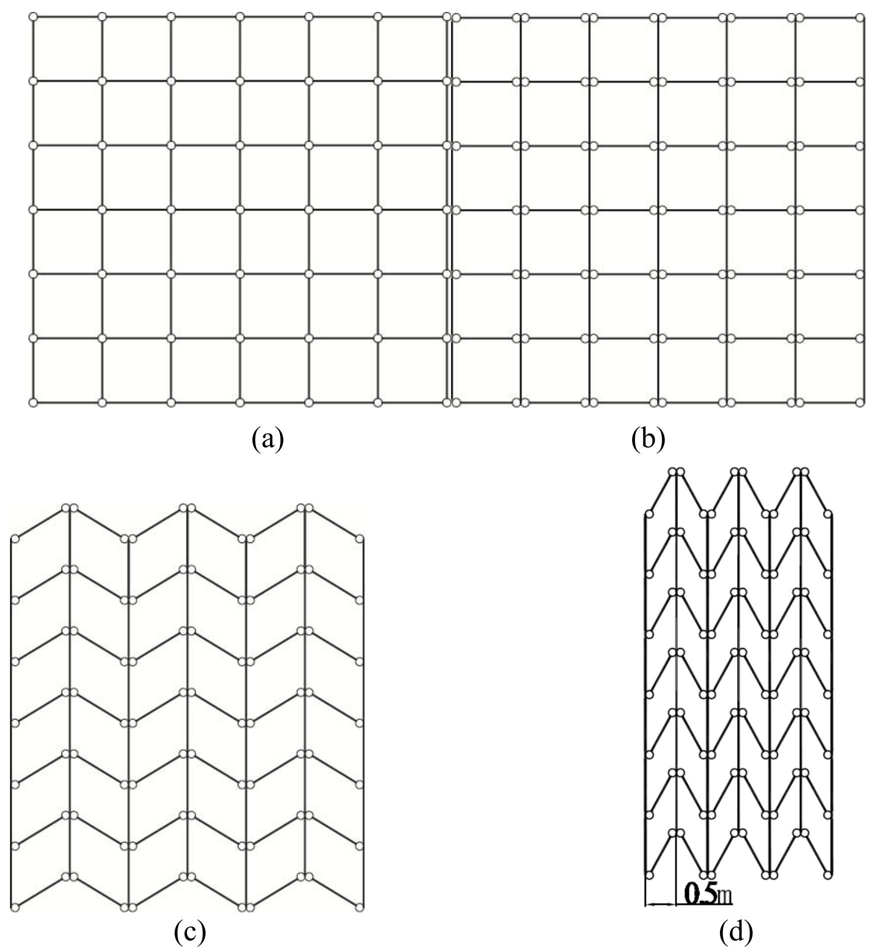
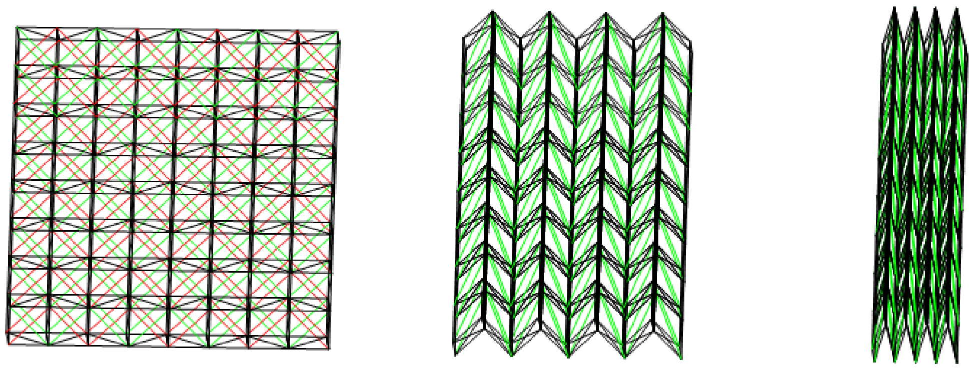


Figure 4. Foldable Truss Structures



(a) Fully open configuration (b) Semi-open configuration (c) Fully closed configuration

Figure 5. 3-D View of the Foldable Truss Structure
(Active Cables not given in Figure 5(b) and 5(c) to Avoid Confusion)

Figure 4(a) presents the plan view of a foldable truss structure formed by the arrangement of spatial foldable units in the horizontal and vertical directions. It can be shown that this truss has high degrees of freedom of this foldable. Therefore, in order to reduce the degree-of-freedom and the complexity of joints, the elements in a row are revised as a non-movable truss shown in Figure 4(b). The foldable truss roof can be then moved in the ways as shown in Figure 4(c) and (d). It should be noted that the distance between two adjacent planar trusses of the structure shown in Figure 4(d) is 0.5m. This is because the two trusses cannot be totally overlapped due to the actual dimensions of bars and joints. Three-dimensional views of the foldable truss structure during the folding are shown in Figure 5, where active cables are in red and passive cables in green.

3. MECHANICAL BEHAVIOR

The mechanical behavior of the foldable truss structure in the fully deployable configuration is studied using ABAQUS, a commercial finite element software package. Main members of the structure are steel bars of yield stress 345 MPa and Young's modulus 2.1×10^5 MPa, which are modeled by beam element B31 with a pipe section of 90mm×4mm (outer diameter × wall thickness). The cables, which are designed to carry tension only, are modeled by truss element T3D2 with a cable radius of 6 mm, yield stress and Young's modulus 1.95×10^5 MPa. The initial stress of cables is 100 MPa. The span, length and height of the grid structure is 24 m in X direction, 24 m in Y direction, 3m in Z direction respectively with a 3 m spacing between two adjacent planar trusses. The boundary condition is given in Figure 6, where supports are arranged every 6 m to provide the vertical and horizontal constraints. Additional horizontal constraints in the X direction are added to the corner nodes of the structure. Obviously, the open or folding of the structure occurs when the horizontal constraints in the X direction of corner nodes are released.

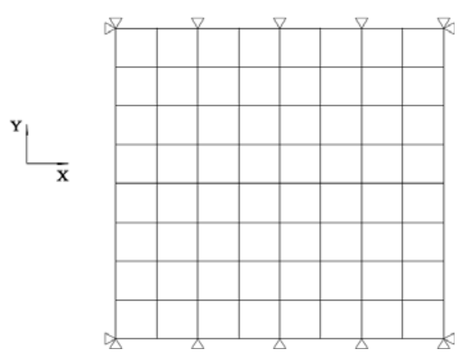


Figure 6. The Support Conditions in the Fully Deployable Configuration

The self-weight of all bars and cables are calculated by the software. The dead load consists of a self-weight of 0.5 kN/m^2 for the roof and the live load is applied to the top surface of the structure in the vertical direction with a magnitude of 0.5 kN/m^2.

The ultimate capacity of some reticulated grid structures can be overestimated if only geometrical non-linearity is considered. It is significantly influenced by the material non-linearity. Therefore, as a new type of foldable reticulated grid structure model, both the material non-linearity and geometrically non-linearity are taken into account in the ultimate capacity analysis of foldable truss structures. The Newton-Raphson method is used to obtain the total load-displacement equilibrium path.

Due to large axial compressive stresses in bars of the grid structure, the load capacity may be greatly affected by the member buckling. Therefore, two models are considered in this paper. One is analyzed without considering the non-linearity due to member buckling while the other one is a non-linear model by meshing a bar into six beam elements. Load-displacement curves of the two models are shown in Figure 7, where the load factor *F* is defined as

$$F = \frac{\text{Imposed load}}{1.2 \times \text{dead load} + 1.4 \times \text{live load}} \quad (2)$$

The load is plotted against the displacement of a node chosen in the area of maximum deformation.

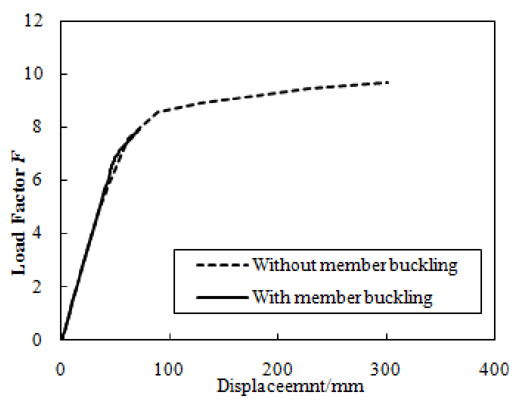


Figure 7. Load factor vs. Displacement for Different Models

The results show that the ultimate capacity of the foldable truss structures without considering the non-linearity due to member buckling is larger than that of the model considering member buckling (where the failure load factor is about 22.4% higher and the maximal displacement is about 314.8% higher). It can be seen that the initial segment of the load-displacement curve is nearly straight due to the slight material and geometrical non-linearity. Moreover, curves for the two different models are almost coinciding. With further application of load, the member suddenly buckles, leading to the ductility of the model, which is considering member buckling, is poor. Therefore, it can be concluded that the result is inaccurate without considering member buckling.

4. EFFECTS OF IMPERFECTIONS ON THE STRUCTURAL BEHAVIOR

The inaccuracy from construction and installation of a structure may have a significant influence on the structural behavior [35]. Thus it is of great importance to study the effect of imperfections on the ultimate capacity. Several methods are available for analyzing the geometrical imperfections, such as the random imperfection mode method, the consistent imperfection mode method, etc. In the latter method, the imperfection distribution is assumed to be consistent with the first buckling mode of the structure. Many researchers have extended the idea of this method to consider the imperfection distribution that is consistent with other deflected shapes of the structure, such as the eigenvalue buckling modes and nonlinear buckling shapes.

It is obvious from the calculation that the first 30 eigenmodes of the foldable truss structure are local buckling. Therefore, imperfections introduced to the structure based on eigenvalue buckling modes almost have no effects on the structural ultimate capacity. Then the non-linear buckling mode, which is shown in Figure 8, is used as the initial imperfection. It can also be found that for prestressed spatial structures, imperfections imposed according to buckling modes may increase the structural failure load if the imperfection is added in the improper direction [36]. Therefore, imperfections are imposed in both positive and negative directions in this paper and parametric analysis on the size of imperfections is carried out.

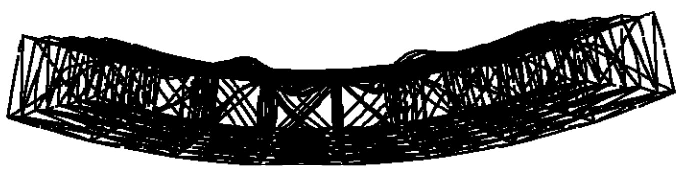
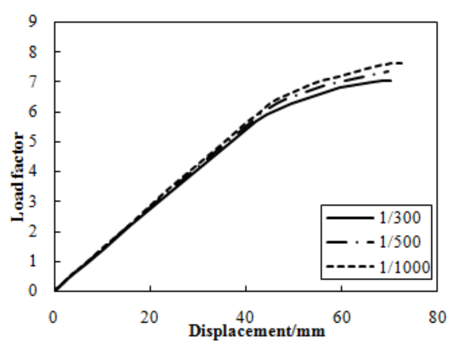
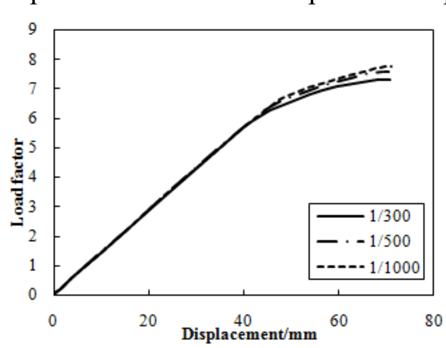


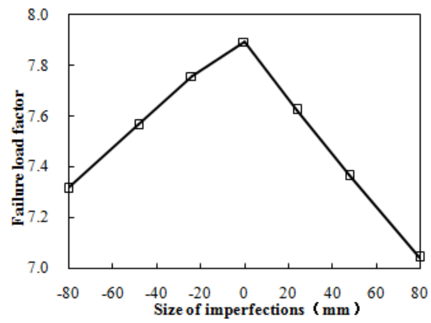
Figure 8. Non-linear Buckling Mode



(a) Load-displacement curves with positive imperfections



(b) Load-displacement curves with negative imperfections



(c) Failure load factors with different sizes of imperfections

Figure 9. Effects of Imperfections on Structural Behavior

The load-displacement curves are given in Figures. 9(a) and 9(b) while Figure 9(c) shows the comparison of the failure load factor with different size of imperfections. It can be seen that the structural stiffness and failure loads reduce slightly with the increase of imperfection and the effect of positive imperfections is more significant than that of negative imperfections.

5. MOVEMENT PROCESS ANALYSIS

It can be seen from Figure 5 that the deployment of the foldable truss structure is implemented by the relative motion between adjacent planar trusses. The release of constraints in the X direction of nodes at one side end will transfer the structure into a mechanism. Then the system moves after the application of the displacement in the Y direction on the interval planar trusses. Considering dimensions of members and joints, the two adjacent planar trusses cannot totally overlap. The maximal displacement in the Y direction of the interval planar truss is 2.9 m while the distance between two adjacent trusses is 0.77 m in fully open configuration.

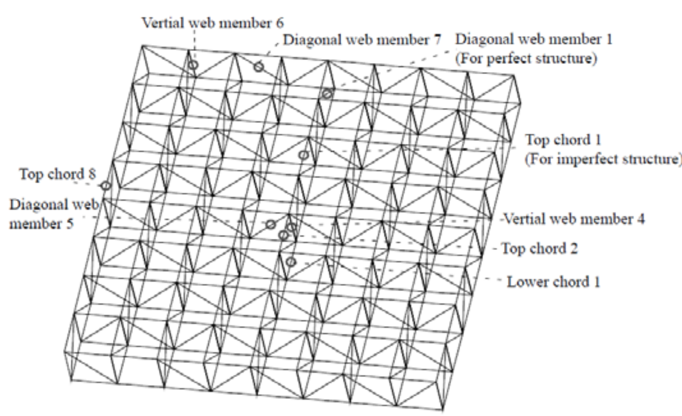


Figure 10. Typical Element Number

In order to investigate the stress distribution of the foldable truss roof during the movement, 8 typical steel bars are chosen as shown in Figure 10. The stresses of these bar elements are given in Figure11. The Mises stress contours at different time during the motion is also shown in Figure 12. It can be seen from these figures that the structure is in fully close configuration when there is no displacement in the Y direction over the interval planar trusses. The maximal member stresses at this time is lower than 40 MPa. During the movement, the member stress of some bars increases while the remaining decreases, but all of them vary slightly. At the end of the deployment, the maximal element stress is 31.86 MPa. Overall the element stresses in the movement of foldable truss structures have changed little.

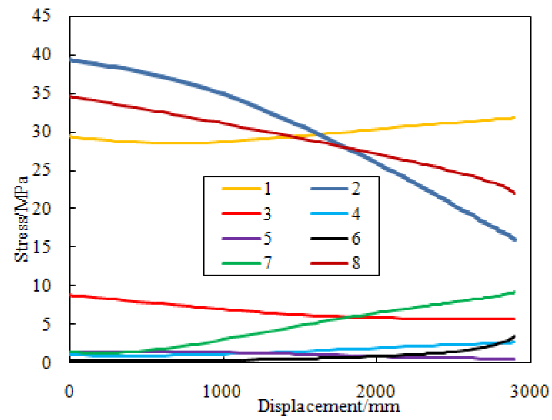


Figure 11. Element Stresses during the Motion

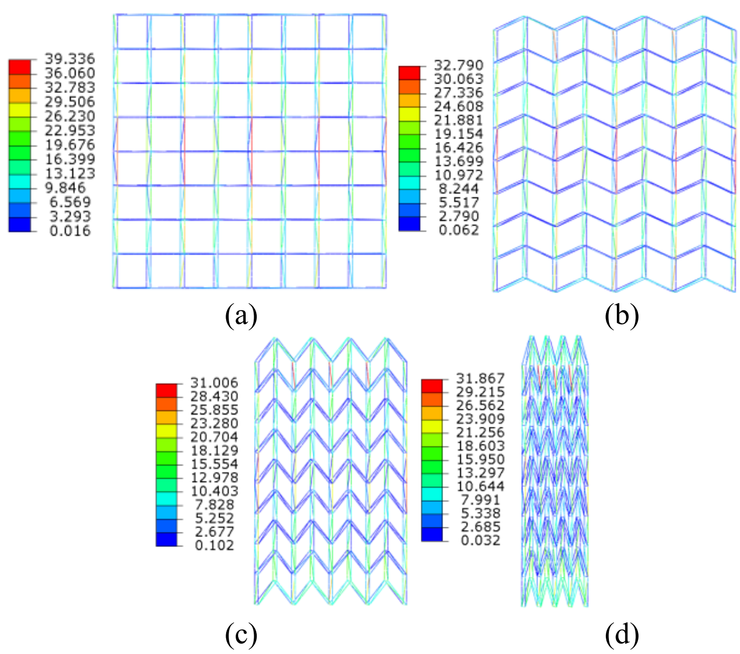
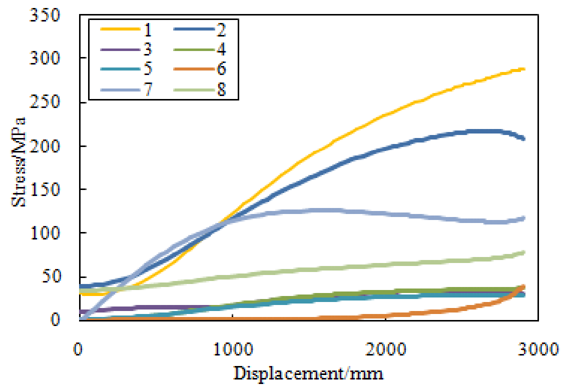


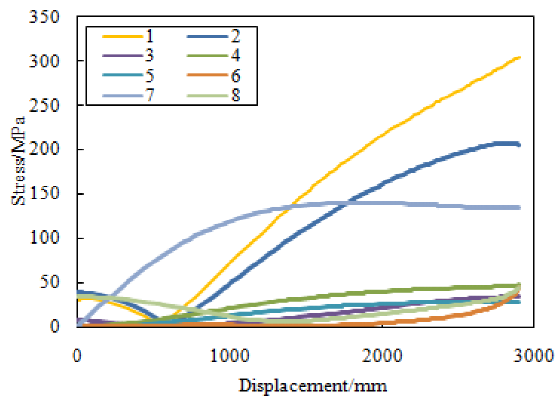
Figure 12. Mises Stress Contour at Different Time (MPa)

6. EFFECTS OF IMPERFECTIONS ON THE MOVING PROCESS

The influence of imperfections on structural behaviour during the motion is investigated in this section. The nonlinear buckling mode [37-41] is chosen as the imperfection distribution for the consistent imperfection mode method. Both positive imperfection and negative imperfection are applied. The stress displacement curves are shown in Figure 13. Note the maximal size of imperfection is assumed to be $L/300$, where L is the span of the structure.



(a) Structures with positive imperfections



(b) Structures with negative imperfections

Figure 13. Element Stresses of Imperfect Structures during the Motion

For structures with positive initial imperfections in motion, it is obvious from Figure 13(a) that element stresses in most members have a significant rise and the maximal stress is 288 MPa at the end of the deployment. In the meantime, the overall stress trend of structures with negative imperfections, given in Figure 13(b), is consistent with that of positive imperfections. The maximal member stress after the deployment is 305 MPa, close to the yield stress of steel member. The member with the maximal stress is located in the region with the largest initial imperfection.

In order to better understand the structural behavior during the motion, Figure 14 presents the Mises stress contour of the structure with positive imperfections at different time. It can be seen that imperfections have a significant influence on the movement of foldable truss structures. Therefore, it is essential to study the effect of sizes of initial imperfections on the movement of the structure. As shown in Figure 15, the effect of different maximal value of initial imperfections with *L*/300, *L*/500 and *L*/1000 on the maximal element stress of the foldable truss structure during the deployment are considered. Obviously, larger initial imperfections will lead to higher maximal member stress. Negative imperfections, however, have a more sensitive effect on the maximal member stress.

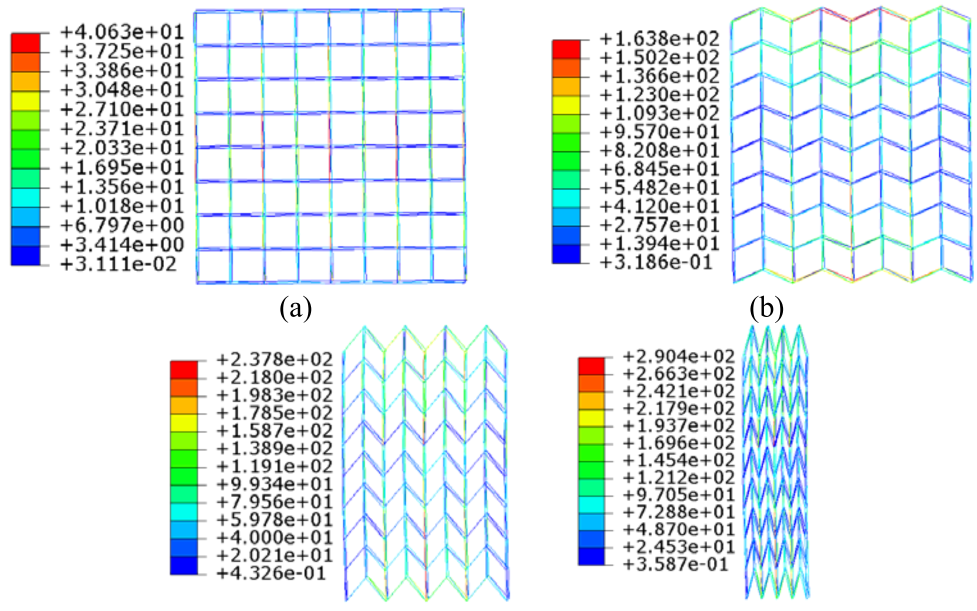


Figure 14. Mises Stress Contour of Positive Imperfect Structures at Different Time (MPa)

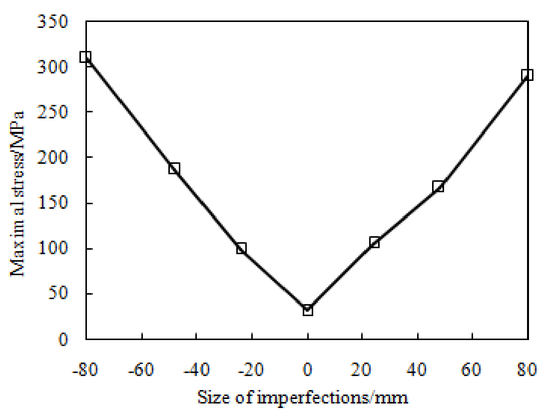


Figure 15. Effects of the Size of Imperfections on Moving Process

7. CONCLUSIONS

In this paper, based on the investigation of the planar four-bar linkage, a type of foldable truss structures are proposed as a retractable roof. Then the mechanical behaviors of the roof system in the fully close configuration and the moving process are studied. From analyses in this paper, conclusions can be drawn as:

(1) The member buckling has a great effect on the ultimate capacity of foldable truss structures. The failure load of the structure with consideration of member buckling is smaller than that regardless of member buckling. Furthermore, the ductility of the structure considering member buckling is poor because of the sudden buckling of bars.

(2) With the increase of imperfections, the structural stiffness and the failure load of the system decrease slightly. Therefore, the effect of imperfections on the ultimate capacity of the foldable structure at the fully closed state is limited.

(3) The variation of member stresses of ideal structures during the folding of the system is small. However, when initial imperfections are introduced, the maximal member stress during the motion increases almost close to the yield stress.

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