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TEACHING-LEARNING BASED OPTIMIZATION METHOD CONSIDERING BUCKLING AND SLENDERNESS RESTRICTION FOR SPACE TRUSSES

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ABSTRACT

The structural performance of a building is a function of several parameters and constraints whose association may offer non unique solutions which, however, meet the design requirements. Therefore, an optimization routine is needed to determine the best solution within the set of available alternatives. In this study, the TLBO method was implemented for weight-based optimization of space trusses. The algorithm applies restrictions related to the critical buckling load as well as the slenderness ratio, which are the basis to obtain reliable and realistic results. To assess the capability of the TLBO method, two reference cases and a transmission tower are subjected to the optimization analysis. In the transmission tower analysis, however, a more realistic approach is adopted as it also considers, through a safety factor, the plastic behavior in the critical buckling load constraint. With no optimization, the ideal weight increases by 101.36% when the critical buckling load is considered in the first two cases, which is consistent with the expected behavior. If the slenderness of the elements is also restricted, the ideal weight now rises by 300.78% from the original case and by 99.04% from the case where only the critical buckling load restriction is applied. Now, considering the critical buckling load and slenderness restriction with the TLBO method applied, a 18.28% reduction in the ideal weight is verified. In fact, the proposed optimization procedure converged to a better solution than that of the reference study, which is based on the genetic algorithms method.

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1. Introduction

The design process requires a series of steps in order to deliver an outcome which is efficient regarding material consumption and cost, and, along with it, meets the design and safety standards. Optimization efforts are generally applied to identify the most suitable solution to the problem from a wide set of design alternatives subjected to restrictions and constraints [1]. The solution can be obtained by minimizing or maximizing an objective function defined according to the governing parameters and imposed restrictions. The optimization routine can be driven by diverse leading and, sometimes, competing fields, so that the objective function can assume distinct formats depending on the driving parameter, e.g., weight, displacement, natural frequency, tension, buckling, etc.

Currently, there is a vast collection of optimization methods applicable to a multitude of problems, especially the nature-inspired meta-heuristic methods. Most well-known and explored processes are based on the genetic algorithms (GA), as proposed by Holland [2]. It belongs to a wider category of evolutionary algorithms which are based on the theory of evolution presented by Darwin, i.e., that evolution occurs through the selection of the most suitable individual. Holland [2] points out that methods such as GA, although effective, require many control parameters, whose ideal values are difficult to determine. Addressing this limitation, Rao et al. [3] presented the teaching-learning-based optimization (TLBO) algorithm, which emulates the teaching interaction between teacher and student, and seeks to solve problems and obtain results with low computational cost and high consistency. Camp and Farshchin [4] modified the method to allow its application to the optimization of space trusses and verified that it was able to provide better results when compared to other population-based techniques. The TLBO method presents attractive features such as easy implementation, versatility, does not require design parameters to direct the search, and has low computational cost.

Lattice structures are widely applied in engineering as they provide excellent resistance/weight ratio. Despite the advantage presented by steel structures, this structural system is prone to collapse due to buckling, as the structure can suddenly or partially collapse without warning [5]. The buckling phenomenon is characteristic of thin sections in compression zones where the overall buckling depends on the attributes of their individual components, which are related with each other influencing the buckling loads [7].

Buckling failure is a critical factor when it comes to slender structures, which is a characteristic feature of metal structures. Caglayan and Yuksel [6] investigated a roof structure failure due to overload caused by the accumulation of snow. Augente and Parisi [7] investigated the failure of a roof structure in the construction phase due to a gust of wind. They pointed out in their results that the steel lattice structure collapsed due to buckling, which propagated causing its total collapse. Episodes like this justify the special attention that must be paid to the buckling restrictions imposed on the structures.

Moreover, there are vast references in the application of optimization methods in the design of steel structures. Safari and Maheri [8] used graph theory for improving the GA and applied it for searching the optimal brace position in 2D steel frames. Fawzia and Fatima [9] presented a parametric optimization methodology for finding the optimum position of steel outrigger systems in high composite buildings, analyzing buildings with 28, 42 and 57 floors. Doğan et al. [10] investigated the effect of the behavior of joints into the design of steel frames using the Hunting Search Algorithm (HSA) to this end. They presented a set of optimum design structures and compared them with the ones obtained by the Particle Swarm Optimization method (PSO) available in the literature. More recently, Gomez et al. [11] applied topology optimization to high buildings under wind stochastic loads. In this study the optimization was accounted using a gradient-based method, demonstrating high efficiency for tall buildings as well.

Haghpanah and Foroughi [12] analyzed the optimization of space trusses using the TLBO method considering critical buckling load restriction values and their percentages to simulate imperfection effects.

In this work, the TLBO was used to optimize space trusses with the objective of minimizing the total weight of the structure. The main focus was to verify the effect of considering critical buckling load and slenderness restrictions on the optimization results. First, a brief theoretical concept is presented followed by a description of the TLBO method including the changes made by Camp and Farshchin [4], which were also considered of this study. Lastly, three case studies are presented to validate and assess the method.

2. Theoretical foundation

2.1. Truss

The truss is a practical and economical structural solution in many applications, e.g., construction of bridges, roof structures, transmission telecommunications towers, among others. By definition, trusses are structures that consists of bars, straight and rigid, subjected to loading at the nodes. Taking into account that truss elements are connected by frictionless hinged joints at their ends, they do not experience bending moments nor shear forces, i.e., truss members only experience axial forces [13]. However, in practice, trusses can have welded or non frictionless bolted joints in addition to imperfections, which can generate bending moments in the structure and induce non-linear behavior [14]. Since truss structures are in general composed of thin elements and can only support a small lateral load, they are liable to the occurrence of buckling [15].

2.2. Buckling

Some structural elements may be subjected to compressive loads which, depending on the length and slenderness of the part, are sufficient to cause a lateral deflection. This phenomenon is referred to as buckling and the maximum load that an element can withstand on the buckling threshold is called critical load (P_{cr}), also known as Euler load [16].

The part will be in stable equilibrium if $P \leq P_{cr}$ and in neutral equilibrium if $P = P_{cr}$. However, if $P \geq P_{cr}$ it is said that the element is in unstable equilibrium. At stable equilibrium, the element remains in the initial position without experiencing lateral displacement. In the case of an ideal column, theoretically, it would remain straight even with the increase of load until its rupture or yielding. However, when the load P_{cr} is reached, a small lateral force F is able to bend the element and it will remain in the bent position even with the removal of the force F. The element will only to return to its initial position if the load in relation to the critical load is reduced [16].

Clearly, the point at which the state changes from stable to unstable is a bifurcation point, which is represented by the critical load. In structural design, the stress state in all elements is expected to fall within the stable equilibrium domain, so that $P \leq P_{cr}$. Since the buckling is related to the flexural strength, the P_{cr} value can be determined by the beam's differential equation, which relates the element's internal bending moment M to its deflected position, given by equation (1):

$$EI\frac{d^2v}{dx^2} = M \tag{1}$$

Applying the considerations for the deflected bars and the moment at the joints equal to zero as a boundary condition, equation (2) yields the following solution:

$$P_{cr} = \frac{\pi^2 EI}{L^2} \tag{2}$$

Thus, the critical buckling stress for the bars can be expressed by equation (3):

$$\sigma_{cr} = \frac{\pi^2 E}{\lambda^2} \tag{3}$$

$$\lambda = \frac{L}{r} \tag{4}$$

where, E is the elastic modulus of the material, I is the moment of inertia of the cross section, L is the length of the column, λ is the slenderness index, r is the radius of rotation of the bar, P_{cr} is the critical buckling load and σ_{cr} is the critical buckling stress. Therefore, the column will buckle around the axis with the lowest moment of inertia [12].

The theory presented so far applies to the ideal case. However, as already mentioned, in practice the structures can present residual stresses caused by geometric imperfections from the manufacturing and construction processes, and eccentricities from the imposed loads [16]. In the case of metallic truss structures, the nonlinear behavior of the model is difficult to be implemented, as it requires the combination of nonlinear sources, the definition of specific parameters, and high computational experience [14]. Thus, safety factors are introduced to compensate these differential effects and provide a more realistic calculation. These factors are specified in the American Institute of Steel Construction (AISC) [17] and are described by equations (5) to (8). The calculation of the safety factor varies according to the value of the slenderness index of the column. These considerations were used in problem 3 of this study.

If $\lambda \geq C$

$$\sigma_{cr} = \frac{12\pi^2 E}{23\lambda^2} \tag{5}$$

If $\lambda \geq C$

$$\sigma_{cr} = \sigma_y\left(1 - \frac{\lambda^2}{2C^2}\right)n \tag{6}$$

$$n = \frac{5}{6} + \frac{3\lambda}{8} - \frac{\lambda^3}{8C^3} \tag{7}$$

$$C = \sqrt{2\pi^2 E/\sigma_y} \tag{8}$$

The Brazilian technical standard that regulates the elaboration of steel structure projects is the NBR 8800 [18], applied to structures formed by rolled or welded steel profiles, of tubular section and with connections made with welding or screws. It is recommended that the slenderness index of the bars should not exceed 300, and that the slenderness index of the compressed bars should not be higher than 200 [18]. These parameters are adopted as the slenderness restrictions in the problems analyzed in this study.

2.3. Problem formulation

Structural optimization procedures, in general, focus on determining the set of parameters that minimizes the weight of the structure. When the geometry of the structure is fixed, the problem focuses on finding the cross sections for the elements in a way that minimizes the weight of the structure while respecting the restrictions. The available cross-sectional areas can be restricted to a range of values, being obtained in a discrete or continuous manner at the beginning of the process. The structure of the optimization problem can be described by the following mathematical formulation.

Objective function:

$$W = \sum_{i=1}^{m} \gamma A_i L_i \tag{9}$$

Constraints:

$$\sigma^L \leq \sigma_i \leq \sigma^U \tag{10}$$

$$\sigma^{cr} \leq \sigma_j \leq 0 \tag{11}$$

$$\delta^L \leq \delta_k \leq \delta^U \tag{12}$$

$$\lambda_i \leq \lambda_{j,t}^U \tag{13}$$

$$A_L \leq A_i \leq A_U \tag{14}$$

$$i \in \{1,2,\dots,m\},\ j \in \{1,2,\dots,m\},\ t \in \{1,2,\dots,m_t\},\ k \in \{1,2,\dots,n\}$$

where W is the weight function, m is the number of components,n is the number of nodes, γ is the specific gravity of the material. For each member i, A_i is the area of the cross section, L_i is the length, σ_i is the stress, and λ_i is the slenderness index. Regarding the constraints, σ^L is the lower limit stress that is equivalent to the maximum compressive stress, σ^U is the upper limit stress that is equivalent to the maximum tensile stress, j designates that the element is under compression andσ^{cr} is the critical buckling stress described in section 2.2. A_L and A_U are the minimum and maximum areas, respectively, for each node $k$analyzed. δ^L and $\delta^U$are the lower and upper displacement limits, which are associated with the negative and positive coordinate displacements, respectively. t indicates the members subjected to traction. $\lambda_j^U$and λ_t^U are the slenderness limits for members under compression and tension, respectively.

In this process, a mechanism known as penalty function is implemented to verify if the model complies with the restrictions. If an arrangement violates the restrictions, a penalty is applied to the result of the objective function for that case. The penalty will be proportional to the value of the sum of the violations. According to Camp [19], the penalized result helps to direct the focus of the research. The amount of penalties is calculated according to the following equations:

The deflection penalty is calculated as:

$$if: \sigma_L \leq \sigma_{cr}\ then\ \sigma_L = \sigma_{cr} \tag{15}$$

$$if: \sigma_L \leq \sigma_i \leq \sigma_U\ then\ \emptyset_\sigma^i = 0 \tag{16}$$

$$if: \sigma_i < \sigma_L\ or\ \sigma_i > \sigma_U\ then\ \emptyset_\sigma^i = \left|\frac{\sigma_i - \sigma_{L,U}}{\sigma_{L,U}}\right| \tag{17}$$

The total voltage penalty for design k is:

$$\emptyset_\sigma^k = \sum_{i=1}^{m} \emptyset_\sigma^i \tag{18}$$

The deflection penalty is calculated as follows:

$$if: \delta_L \leq \delta_{i(x,y,z)} \leq \delta_U\ then\ \emptyset_\delta^i = 0 \tag{19}$$

$$if: \delta_L < \delta_{i(x,y,z)} \; or \; \delta_{i(x,y,z)} > \delta_U \; then \; \emptyset^i_\delta = \left|\frac{\delta_i - \delta_{L,U}}{\delta_{L,U}}\right| \tag{20}$$

The total deflection penalty for design k is:

$$\emptyset^k_\delta = \sum_{i=1}^{m}\left[\emptyset^i_{\delta(x)} + \emptyset^i_{\delta(y)} + \emptyset^i_{\delta(z)}\right] \tag{21}$$

The calculation of the slenderness penalty is:

$$if: \lambda_i \le \lambda^U_{j,t} \; then \; \emptyset^i_\lambda = 0 \tag{22}$$

$$if: \lambda_i > \lambda^U_{j,t} \; then \; \emptyset^i_\lambda = \left|\frac{\lambda_i - \lambda_{U(j,t)}}{\lambda_{U(J,t)}}\right| \tag{23}$$

The total slenderness penalty for design k is:

$$\emptyset^k_\lambda = \sum_{i=1}^{m} \emptyset^i_\lambda \tag{24}$$

The total penalty amount for the truss design k is the result of the sum of the penalties:

$$c^k = \left(1 + \emptyset^k_\sigma + \emptyset^k_\delta + \emptyset^k_\lambda\right)^\varepsilon \tag{25}$$

where ε is the positive penalty exponent, defined as 2 in this work, as also adopted by Camp and Farshchin [4]. Lastly, the penalty amount is applied to the weight determined for the truss design k:

$$F^k = w^k c^k \tag{26}$$

3. Teaching-learning-based optimization

The TLBO method is an optimization method whose main inspiration is the process of teaching and learning. It takes place in a classroom and resembles the relationship between teacher and student, as its ultimate goal is to make students increase their level of knowledge. It is assumed that the teacher is the most instructed individual in the class and will try to share his knowledge with the students so that they increase their level of knowledge. Rao et al [3] showed that the result is proportional to the level of the teacher, so he is able to increase the knowledge of a class from an initial level to a higher level.

In addition to the phase where the teacher is the vector guiding the process, it is understood that students can interact with each other. In this phase, they will be protagonists of the process by sharing knowledge collaboratively, which also contributes to raising the level of knowledge of the class. The evaluation of the performance of each student is carried out through an exam and the result is the grade. The final performance of the class can be described by the normal probability curve.

As previously mentioned, the method is divided into two phases, the teacher phase and the student phase. Next, both phases will be described in accordance with Rao et al [3], including the changes proposed by Camp and Farshchin [4].

3.1. Teacher Phase

In this phase, the method tries to model the influence of the teacher on the students of the class in such a way that the students try to update their knowledge according to the information provided by the teacher. At the teacher's phase, the students will increase their knowledge, that is, all students are influenced by the teacher. In practice, a teacher can only move the level of the classroom to a limited stage which is proportional to his capacity. This process is modeled by the following mathematical expressions:

$$X^k_{new}(j) = X^k_{old}(j) \pm \Delta(j) \tag{27}$$

$$\Delta(j) = T_F * r|M(j) - T(j)| \tag{28}$$

where, $X^k(j)$ is the student's grade k in the discipline j, which is, by analogy, the design variable j of the solution vector k. T_F is the teaching factor, r is a random number in the range between [0 1], $M(j)$ is the class average, and $T(j)$ is the teacher's status. The teaching factor T_F decides the average that will be changed and represents the teacher's ability. Its value can be chosen as 1 or 2, or randomly by the expression:

$$T_F = round[1 + rand(0,1)\{2 - 1\}] \tag{29}$$

The studies by Togan and Mortazavi [20] compared the use of the three possibilities for the value assumed by T_F, i.e., 1, 2 or a randomly generated number (equation 29), concluding that any of the three choices do not interfere in the final optimization result. For this work, the value 2 was used, the same used by Camp and Farshchin [4].

The work of Rao et al [21] and Rao and Patel [22] emphasize that the value of T_F is not a parameter, since an input value is not mandatory, as occurs in genetic algorithms, and because its value can be generated randomly by equation (29).

In this work, the TLBO method modified by Camp and Farshchin [4] was used, which provides or calculates the average of the average of students with weighted average, a principle calculated by Rao et al [3] in an arithmetic way. The M value is determined by the following expression:

$$M(j) = \frac{\sum_{k=1}^{N} \frac{X^k(j)}{F^k}}{\sum_{k=1}^{N} \frac{1}{F^k}} \tag{30}$$

where F^k is the weight of the penalized structure (Eq. (26)), and N is the population number.

The updated $X^k_{new}(j)$ value for each student will be accepted if the update value is better than the old $X^k_{old}(j)$. All values updated at the end of the teacher phase are retained and become input values for the student phase.

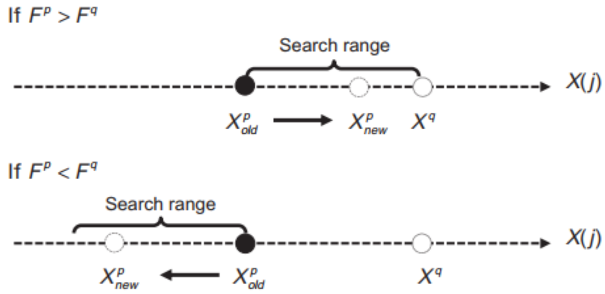

Fig. 1 Learner phase, [4]

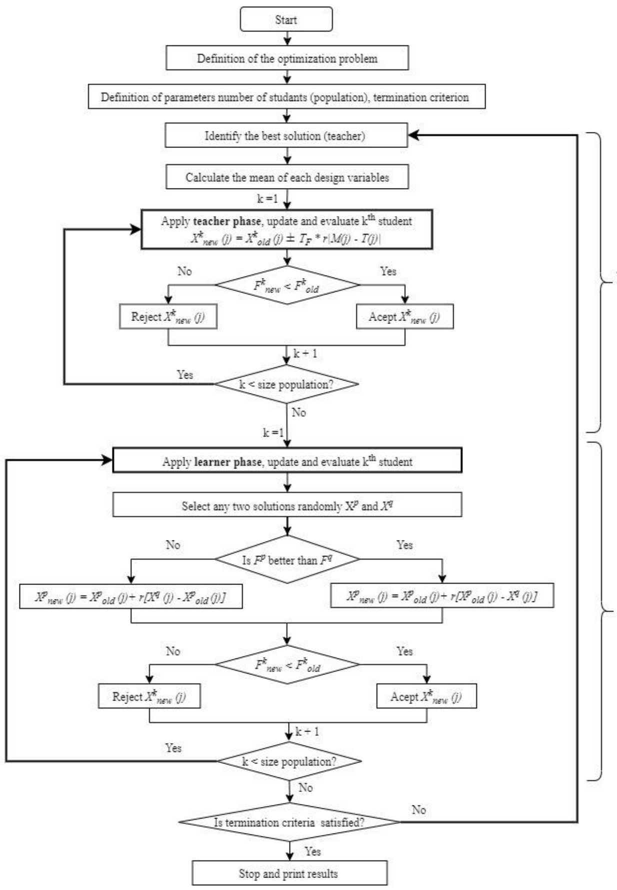

Fig. 2 Flowchart - TLBO Method, [3]

3.2. Learner Phase

This phase corresponds to the stage at which students learn through the interaction between themselves. At this stage, each student in the class randomly interacts with another student. It is understood that a student will learn new things from another who has more knowledge. In this phase, each student is compared with another student in the class. The learning process in this phase occurs as follows:

(a) A student p is selected randomly in the class;
(b) A second student q is selected randomly, with $q \neq p$;
(c) F_p^k and F_q^k are calculated;

$$X_{new}^p(j) = \begin{cases} X_{old}^p(j) + r\left[X_{old}^p(j) - X^q(j)\right] \; if \; F_p^k < F_q^k \\ X_{old}^p(j) + r\left[X^q(j) - X_{old}^p(j)\right] \; if \; F_p^k \geq F_q^k \end{cases} \quad (31)$$

where, *r* is a random number in the range [0.1]. Fig. 1 illustrates the behavior of students in the process.

Fig. 2 presents a flowchart that describes the process of operation of the TLBO method.

3.3. Operation of the method

The computational procedure mimics the process described above, divided into the teacher and student phases. The TLBO method is a population-based method. According to Togan [23], the following analogy can be established between the TLBO and the ideal configuration of the structure: the students in the class represent the candidate configurations, each discipline represents the design variable (the steel section), and its combinations represent the variables of the project.

Next, the procedure for implementing the TLBO method will be described as presented by Rao et al [21].

Initialization of the problem:

Step 1: define the optimization parameters, number of students - population (*Pn*), number of groups - number of design variables (*Dn*), number of generations (*Gn*) and the limits of the design variables described in section 2.3.

The optimization problem is defined by the maximization function *w(x)* subjected to $x = x1, x2, \ldots, xDn$. Where x_i are the design variables that must comply with the imposed restrictions.

Step 2: The population is initialized randomly according to its size, the number of design variables, and the design variables available.

$$population = \begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,D} \\ x_{2,1} & x_{1,1} & \cdots & x_{2,D} \\ \vdots & \vdots & \vdots & \vdots \\ x_{Pn,1} & x_{Pn,2} & \ldots & x_{Pn,D} \end{bmatrix} \begin{matrix} \rightarrow w_1(x) \\ \rightarrow w_2(x) \\ \rightarrow \; \vdots \\ \rightarrow w_{Pn}(x) \end{matrix} \quad (32)$$

Step 3: teacher phase.

First, the weighted average grade of the population is calculated in rows and this will yield the class average grade in each discipline resulting in a $M_D = [m_1, m_2, \ldots, m_{Dn}]$. The best solution will interact with a teacher and try to evolve the students' knowledge towards their own level of knowledge. At this stage, the knowledge of each student will be influenced by the teacher (equation 27). Updated students are accepted if their knowledge is improved.

Step 4: learner phase.

The code receives the updated population vector (students) from the teacher phase. Students interact with each other randomly, as described in the student phase (section 3.2).

Step 5: stop criteria.

There two stop criteria, i.e., when the maximum iteration limit is reached at 200 or when the restrictions imposed by the optimization problem is met, see section 2.3. In this method, a penalty is applied to solutions that exceed the imposed restrictions (see section 3). If it does not meet the stop criteria, the process returns to step 3.

All the work was done using Matlab commercial software. The Matlab is a well known software that works to solve problems based on matrix analysis. The structural analysis process was implemented using the finite element method (FEM). The implemented program followed the following procedure: First, a code was created to insert the data, coordinates, incidence, loads, boundary conditions, division of groups and restrictions. Second, the structural analysis by the FEM is carried out, which includes calculation of the stiffness matrix, displacement, tension and penalty. This process works in parallel with the TLBO method, teacher and student phases.

4. Results and discussions

Three examples are addressed for discussion, where the first two are reference examples and the third is a tower approached by Galante and Onate [24] and Rajeev and Krishnamoorthy [25]. In the first two examples, the obtained results are compared against those published by Camp and Farshchin [4] to validate the code. The first is a 25-bar truss divided into 8 groups, the second a 72-bar truss divided into 16 groups, and the third a 160-bar tower divided into 16 groups. The population of 75 students was adopted, which, according to Camp and Farshchin [4], is an ideal value to obtain good results with low computational cost. The TLBO algorithm was implemented to perform a maximum of 200 iterations if convergence is not reached by then, which is the same criterion used by Camp and Farshchin [4]. The results are presented in the form of cross sections and ideal weight.

In addition, for each case, the optimization was repeated one hundred times in order to verify the uniformity of the results. The analyses studied in this paper were carried out in a laptop model I14-2640P, with Core i5 processor and 8GB RAM.

4.1. Problem 1: 25-bar tower

The 25-bar truss tower is shown in Fig. 3 and the applied loads are shown in table 1. For the optimization problem, the following parameters were assumed: elastic modulus of 68.95 GPa, density 2767.99 kg/m³, maximum nodal displacement of 8.89 mm and maximum permissible stress of 275.79 MPa. More details about the structure can be found in Camp and Farshchin [4].

To validate the code, the cross sections given by Camp and Farshchin [4] were used, ranging from 0.645 cm² to 21.935 cm², with an increase of 0.645 cm². For application of the buckling conditions, 50 corner profiles with equal leg angles were used, as provided in the catalog of the company Gerdau - Catalog of equal leg angles - with cross sectional area varying from 0.7 cm² to 73.81 cm².

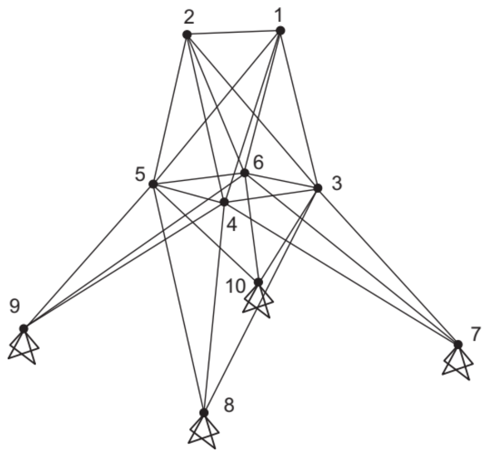

Fig. 3 25-bar tower, [4]

Table 1
Applied Loads on the 25-bar tower, [12]

Node	Fx (kN)	Fy (kN)	Fz (kN)
1	4.448	-44.482	-44.482
2	0	-44.482	-44.482
3	2.224	0	0
6	2.669	0	0

The optimization results are shown in table 2. To validate the code, the case addressed by Camp and Farshchin [4] was applied without applying the buckling restrictions. The present work converged to the same total weight of the structure obtained by Camp and Farshchin [4], i.e., 219.93 kg. These results can be seen in column 3 and 4 of Table 2.

After validating the code, three situations were analyzed for the problem, case 1: without applying the buckling and slenderness restriction, case 2: applying the critical buckling load restriction, and case 3: considering the critical load restriction including buckling and slenderness. For case 1, the TLBO obtained a value of 212.54 kg. For case 2, the result converged to 343.12kg, which represents an increase of 56.02% compared to case 1. For case 3, the ideal weight obtained was 353.70 kg, i.e., an increase of 60.83% compared to case 1.

For case 1, 15 bars were compressed, of which 12 presented stresses greater than the critical buckling load and 10 reached slenderness above 200. 5 of the tensioned bars presented slenderness above 300, in both the slenderness reaches 1437.8. For case 2, there was a reduction in the slenderness index of the bars, 12 were compressed, of which 4 showed slenderness above 200 reaching 250.66, and 3 of the compressed bars have slenderness above 300, reaching 762.

Table 2
TLBO results for the 25-bar tower

Variables		Cross-sectional areas (cm²)				
Group	Members	Camp and Farshchin [4]	Validation	Present		
				Normal	With σ_{cr}	With $\sigma_{cr} + \lambda$
1	1	0.645	0.645	0.700	0.700	1.930
2	2-5	1.935	1.935	0.700	12.510	12.510
3	6-9	21.935	21.935	24.190	15.730	15.730
4	10,11	0.645	0.645	0.700	0.700	1.930
5	12-13	13.548	13.548	13.500	2.320	4.580
6	14-17	6.452	6.452	5.220	15.480	15.730
7	18-21	3.226	3.226	0.900	15.730	19.500
8	22-25	21.935	21.935	24.190	23.290	19.500
Weight (kg)		219.924	219.926	212.539	343.118	353.701
Average weight (kg)		219.951	219.980	212.791	362.875	385.537
N° analyses		4910	6664	6623	12004	13674
Average time (s)		-	32.436	31.987	54.803	61.857

4.2. Problem 2: 72-bar tower

Fig. 4 illustrates the 72-bar truss configuration. For optimization, the following data were considered: maximum allowable stress of 172.37 MPa, maximum nodal displacement of 6.35 mm, material density 2768 kg/m³, material elastic modulus of 68.95 GPa and loading at node 17 with 22.241 kN, 22.241 kN and -22.241 kN in the x, y and z directions, respectively.

For this case, the code was validated using a discrete variation in cross-sectional area with an interval ranging from 0.645 cm² to 19.355 cm² at an increment of 0.00645 cm². Camp and Farshchin [4] used the same interval with continuous variation. In this case, the method converged to an ideal weight of 167.698 kg, less than the value of [4], i.e.,172.198 kg. The optimization results are described in table 3.

For this problem, the same analysis performed in the previous problem was repeated here, and the same profiles were also used. For case 1, the method obtained a value of 169.69 kg, a slight reduction in weight compared to the validation case, which was due to the use of smaller profile areas. For case 2, the result converged to 341.68 kg, which represents an increase of 101.36% compared to case 1. For case 3, a significant increase occurred as the solution converged to the weight of 680.07 kg, representing an increase of 300.78% in relation to case 1.

For case 1, the tower contained 36 bars under compression, 29 exceeding the critical buckling load, 30 of the compressed bars reached slenderness above 200 and 34 compressed bars presented slenderness above 300, in both cases the slenderness reaches 1724.2. For case 2, the tower started to present 41 compressed bars, of which 27 reached slenderness above 200 reaching 952.5, and 10 of the compressed bars showed slenderness above 300, continuing to reach the value of 1724.2

Table 3
TLBO results for the 72-bar tower

Variables		Cross-sectional areas (cm²)				
Group	Members	Camp and Farshchin [4]	Validation	Present		
				Without σ_{cr}	With σ_{cr}	With $\sigma_{cr} + \lambda$
1	1-4	12.1335	12.000	11.480	7.030	7.030
2	5-12	3.3174	3.226	3.420	7.030	12.510
3	13-16	0.6452	0.645	0.700	1.930	10.900
4	17,18	0.6452	0.645	0.700	0.700	7.030
5	19-22	8.2006	8.194	7.670	5.800	7.030
6	23-30	3.3232	3.290	3.100	7.030	10.900
7	31-34	0.6452	0.645	0.700	1.930	15.480
8	35,36	0.6452	0.645	0.700	2.320	10.900
9	37-40	3.4303	3.226	3.420	4.580	4.000
10	41-48	3.3123	3.226	3.420	7.030	15.480
11	49-52	0.6452	0.645	0.700	0.900	5.800
12	53,54	0.6452	0.645	0.700	2.710	10.900
13	55-58	1.0097	0.645	0.700	4.580	4.580
14	59-68	3.5026	3.355	3.420	9.290	10.900
15	67-70	2.6329	2.516	2.770	5.800	12.510
16	71,72	3.6987	3.613	3.420	10.900	19.500
Weight (kg)		172.198	167.698	169.688	341.677	680.070
Avarage weight (kg)		172.256	167.732	169.887	369.003	892.896
N° Analyses		21542	12989	10350	21692	16949
Avarage time (s)		-	185.542	141.247	285.149	213.472

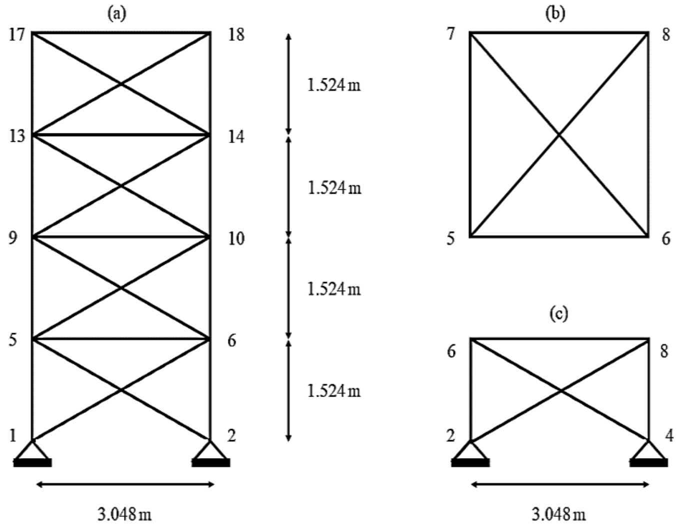

Fig. 4 72-bar tower: a) front view b) cross section c) side view, [8]

4.3. Problem 3: 160-bar tower

The 160-bar truss tower is illustrated in Fig. 5. This truss was solved by Rajeev and Krishnamoorthy [25], using GA to minimize weight without buckling. Galante and Onate [24] also analyzed the same structure using GA, optimizing it for the case where buckling and slenderness restrictions are applied. Both authors used 32 variations of cross-sectional areas taken from the AISC manual, which are also used in this problem. Galante and Onate [24] divided the structure into 16 groups, the author proposed this division in order to obtain a structure with little variation in profiles in order to find a structure that facilitates construction.

In this problem, the same values proposed by Rajeev and Krishnamoorthy [25] and Galante and Onate [24] were considered: elastic modulus of 208 GPa material, steel density of 7850 kg / m³, allowable stress of 147.15 MPa and maximum nodal displacement of 0.1 m. The assumed loads are shown in table 4. The structure was divided into 16 groups, as adopted by Galante and Onate [24]. More details about the structure can be found in Galante and Onate [24].

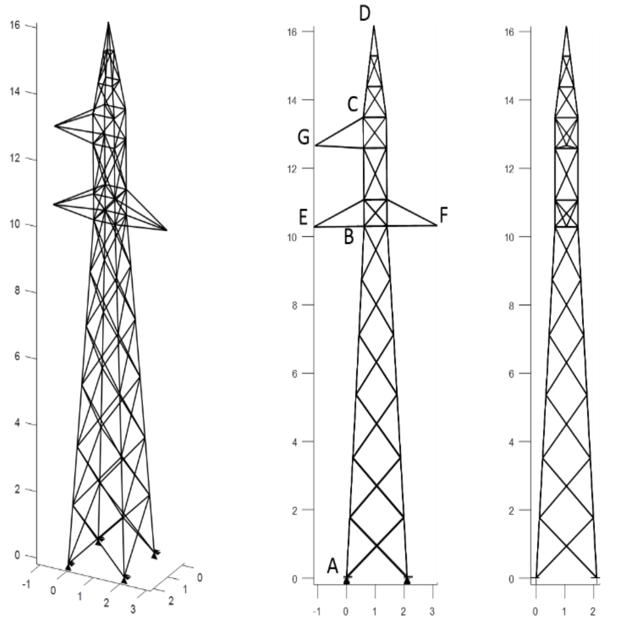

Fig. 5 160-bar tower

Table 4
Loading 160-bar tower, [20]

Node	Fx (N)	Fy (N)	Fz (N)
D	-10702.7	0	-5356.26
E	-10702.7	0	-5356.26
F	-9770.76	0	-5356.26
G	-8515.08	0	-4816.71

The optimization results can be seen in table 5. For the case when buckling and slenderness are disregarded, the TLBO obtained the ideal weight of 577.22 kg, which was between the values found by Rajeev and Krishnamoorthy [25] and Galante and Onate [24], i.e., 662.3 kg and 547.4 kg, respectively.

Table 5
Results for the 160-bar tower

Variables		**Cross-sectional areas (cm²)**			
Group	**Members**	**Galante and Onate [24] Without buckling**	**Present Without buckling**	**Galante and Onate [24] With $\sigma_{cr} + \lambda$**	**Present With $\sigma_{cr} + \lambda$**
1	1-24	-	9.290	12.5161	9.290
2	25-36	-	3.632	7.6774	4.613
3	37-48	-	2.800	4.6129	1.716
4	49-56	-	1.510	4.6129	7.032
5	57-64	-	1.510	4.6129	5.819
6	65-72	-	1.510	4.6129	7.032
7	73-80	-	1.510	4.6129	7.032
8	81-88	-	1.510	4.6129	3.123
9	89-96	-	1.510	4.6129	5.819
10	97-104	-	1.510	4.6129	2.800
11	105-112	-	1.510	4.6129	3.123
12	113-120	-	1.510	4.6129	3.123
13	121-128	-	1.510	4.6129	1.510
14	129-136	-	1.510	4.6129	1.510
15	137-148	-	1.510	4.6129	1.510
16	149-160	-	1.510	4.6129	4.613
Weight (kg)		547.400	577.217	1293.800	1057.266
Avarage weight (kg)		-	578.196	-	1180.162
N° Analyses		-	5196	-	19384
Avarage time (s)		-	135.615	-	531.863

For the case considering the critical buckling load restrictions described in section 2.2, as used by Galante and Onate [24], the TLBO reached the ideal weight of 1057.27 kg, which is below the value of 1293.8 kg reached by Galante and Onate [24] using GA, representing a reduction of 18.28%. Similarly to the two previous problems, it is possible to notice that the ideal structure undergoes a considerable increase in the ideal weight when considering the critical load restrictions imposed by buckling and slenderness.

Analyzing the behavior of the structure, it can be seen that, in the first case, without considering the restrictions of critical load of buckling and slenderness, 80 bars are subject to compressive stress, of which 27 have a compressive stress greater than the critical buckling stress specified in AISC manual (Eqs. (5) to (8)). Analyzing all the compressed bars, 43 exceed the slenderness limit of 200. In the case of compressed bars, 35 exceed the slenderness limit of 300. In both cases, the slenderness reaches 532.62. Therefore, the structure obtained when disregarding the buckling effect is not a proper structural load system, as it would not meet normative restrictions and would probably collapse [24].

5. Conclusion

Through this work, it is verified that the buckling effect plays a significant role in the optimization of truss structures, as it provides more realistic results due to the increased ideal weight of the structure. As indicated in section 2.2, depending on the geometric characteristics and on the modulus of elasticity, the critical buckling load may be less than the allowable stress for the material, therefore, larger sections are required to withstand the acting stresses, which justifies the increase in ideal weight.

It can be seen that, by restricting the critical buckling load, this resulted in a reduction in the slenderness index of some compressed elements, which is justified by the fact that the critical buckling load, in some bars, is below the allowable stress for the material, therefore requiring more robust sections. Thus, by limiting the slenderness of the parts, the standard regulation further restricts the support capacity of the structure, which is equivalent to the application of an additional safety factor.

For the optimized cases, in which the critical buckling load and slenderness restrictions were not considered, several elements violated the critical conditions, which would probably cause the collapse of these structures. This fact confirms Galant and Onate's conclusion, which states that meeting the critical load restrictions imposed by the buckling and slenderness index is fundamental for a safe and more realistic project [24].

Computational codes are available under request (only for academic purposes). Please contact the corresponding author for more information.

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