

INELASTIC STABILITY ANALYSIS FOR FRAMED STRUCTURES SUBJECTED TO NONCONSERVATIVE FORCES

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ABSTRACT: In this study, the inelastic system buckling analysis is developed to evaluate the buckling loads and the effective length factors for columns in steel frames subjected to nonconservative forces. For this purpose, the finite element model for the nonconservative system is presented based on an extended Hamilton principle. The traditional elastic and inelastic stability analyses are briefly introduced for the conservative systems. Then evaluation procedure for the critical value of buckling load and its corresponding effective length factor for the nonconservative system are proposed based on the inelastic system buckling approach. In numerical examples, the buckling loads and the effective length factors for the inelastic conservative or nonconservative steel frames are evaluated and compared with results by other researchers. Particularly, the effects of various parameters such as the nonconservativeness parameter, the stiffness and span ratios of beam to column of frames on the inelastic stability behavior of nonconservative systems are newly addressed.

Keywords: Effective length factor, Inelastic column, Nonconservative force, Finite element method

1. INTRODUCTION

For decades, many researchers have explored various approaches for assessing column and frame stability in the design of steel structures. Among these approaches, the stability design method based on the effective length factor (K -factor) was quite a common practice to engineers for many years.

Up to the present, various methods have been reported for the evaluation of K -factor and the most widely accepted method is the alignment chart method that was proposed by Julian and Lawrence [1]. And now, various design codes such as AISC [2], ACI [3], and AASHTO [4] adopt this method. The isolated subassembly approach, as an alternative method, has also been applied extensively following the early works of Galambos [5] and adopted in specifications such as DIN 18800 [6] and Eurocode 3 [7]. These alignment chart method and isolated subassembly approach are the traditional methods used to assess the rotational end restraints of the isolated column, which represent the stiffness ratio between the column and the beam. However in spite of its popularity, these approaches have major drawback in that they do not properly reflect the interaction effects of neighboring members except for the very closely adjacent columns and beams. Since these approaches are based on several fundamental assumptions, which basically limit the use of these methods only for the idealized cases, they may lead to inadequate estimation of the K -factor when these assumptions are not satisfied. To overcome this drawback, many studies have been carried out. Duan and Chen [8, 9] evaluated the K -factor of the three-story subassemblage of the frame using the same assumptions as the traditional effective length method, but considering the true boundary conditions at far ends of the top and bottom columns. The story-interaction of columns in no-sway frames was studied by Bridge and Fraser [10] and they extended the nomograph for effective lengths of non-sway buckling to a range where the rotational spring had a negative rigidity and proposed an iterative approach to calculate this interaction. Mahini and Seyyedian [11] presented a new iterative approach with a high convergence

rate to improve the effectiveness of the G -factor method developed by Bridge and Fraser [10]. They replaced the explicit forms of the stability functions by more simple rational forms using the curve-fitting principles and converted members with the non-hinged far-end condition to the equivalent hinged far-end ones, considering the rotational stiffness in each case. The improved analytical expressions for K -factors of columns in multi-story sway frames were also proposed by Gantes and Mageirou [12]. They considered all possible rotational and translational boundary conditions at the far end of these members, as well as the eventual presence of axial force.

Another improved approach to evaluate the K -factor is the so called “story-based approach” which accounts for the horizontal interaction between columns in a story. In the Commentary of the AISC [2], two methods of determining the story-based K -factor, namely the story buckling method [13] and the story stiffness method [14,15], were presented. LeMessurier [14,15] provided comprehensive discussion for story stability behavior and proposed a practical approach for obtaining K -factors to consider the story stability behavior. Also the in-plane buckling characteristics of the unbraced frames under the non-proportional loading adopting the story-based buckling concept were investigated by Xu *et al.* [16]. To get over the difficulty associated with the non-proportional loading, the problem of the lateral buckling of unbraced frames was expressed as a minimization problem subjected to stability constraints. Liu and Xu [17] presented a practical method for evaluating K -factors for columns in multi-story unbraced frames based on the story-based buckling concept, which decomposes a multi-story frame into a series of single-story partially-restrained frames.

The stability of the column in the frame structures, although often expressed as a stability problem restricted to an individual column, can be regarded as a problem in related with the whole stability of the structures by considering interactions between all members. Thus the column design is a system-related problem, not an individual member-restricted problem. Özmen and Girgin [18] developed the method for determining K -factors of columns in multi-story frames based on computing on approximate values for system buckling load by using the results of a fictitious lateral loading. Girgin *et al.* [19] applied their previous method [18] to irregular frames and obtained good results with errors less than 5% for all examples considered. Geschwindner [20] reviewed a wide range of analytical approaches including the elastic system buckling analysis and Roddis *et al.* [21] pointed out the practical limitations of the alignment chart method using the elastic system buckling analysis. Despite significant achievements, all of these studies were devoted only to the elastic stability of the overall frame systems and did not consider the inelastic effects of columns.

To account for the effects of inelastic behavior on evaluation of K -factor, several researchers have studied the inelastic stability design of steel structures. Yura [13] proposed an iterative procedure to determine the K -factor in the inelastic range of column behavior. Choi *et al.* [22] presented the numerical method determining the inelastic effective length of each member in three-dimensional steel frames based on iterative schemes and the stiffness reduction factors considering a fictitious axial force factor. Kim and Lee [23] investigated the influence of the P - Δ effect on the behavior of middle-rise unbraced steel frames using the refined plastic hinge method with an arc length algorithm. Essa [24] developed a design approach, to predict K -factors for columns in unbraced frames, considering the inelastic behavior, semi-rigid connections, far-end conditions, and differentiated stiffness parameters of the connected columns. He used a slope-deflection equation with the column stiffness reduction factors, which were evaluated from the column-strength curve proposed by the Structural Stability Research Council (SSRC) [25]. In AISC [2], inelastic K -factors are determined from the alignment chart method with the stiffness reduction factor applied to elastic modulus in the equation for G -factor. Depending on how the stiffness reduction factors are calculated, they might account for both the reduction in the stiffness of columns due to geometric imperfections and the spread of plasticity due to residual stresses under high compression loading. However, these studies

were based on the alignment chart method, which adopted several simplifying assumptions and restricted to the stability analysis of frames under the conservative forces.

The nonconservative forces are forces whose line of action is affected by the deformation of the system on which they act and some examples of the nonconservative systems are the frame structures, the members of bridges, and aircraft which are subjected to aerodynamic force. The submarines and ships are also subjected to hydrodynamic forces which are nonconservative. The nonconservative forces are present in basically every engineering field such as civil engineering and bioengineering (spinal chord) among others and considerable attention has been devoted to the stability of columns subjected to these nonconservative forces. The first review of this branch of applied mechanics has been made in a book form by Bolotin [26]. Gasparini *et al.* [27] discussed the transition between the stability and instability of a cantilevered beam subjected to the partially follower force. Ryu *et al.* [28] employed the finite element method to perform the numerical analyses on the stability of a cantilevered column subjected to the triangularly distributed subtangential forces. Wang [29] studied the effect of the follower force on the buckling capacities of the beam structure subjected to the concentrated follower force. Imielowski [30] performed the sensitivity analysis of a stepped column under the subtangential force depending on the nonconservativeness parameter and investigated the influence of joint stiffness, variation of segment cross-sections and their length. But these works only dealt with the elastic stability problems of a single column under the nonconservative forces. Lee *et al.* [31] investigated the dynamic stability behavior of the Beck's column resting on elastic foundation by using the finite element method.

From the previously cited references, it can be noted that even though a significant amount of research has been extensively conducted on the evaluation of K -factor for the stability design of frames, there still has been no study reported on the elastic or inelastic instability analyses of steel frames subjected to nonconservative forces. The main purpose of this study is to present a more reliable procedure to determine K -factors of columns in steel frames subjected to nonconservative forces based on the elastic and inelastic system buckling analyses by using the column strength curves. The important points considered in this study are summarized as follows:

1. Based on the extended Hamilton principle, the finite element model is formulated. For this, the generalized displacements are expressed over each element as a linear combination of the one-dimensional Lagrange interpolation function for the axial displacement and the cubic Hermite interpolation function for the lateral displacement.
2. Traditional elastic and inelastic stability analyses such as the isolated subassembly approach and the story-based one are briefly presented for the conservative systems.
3. Inelastic system buckling analysis procedure using the AISC-LRFD and Eurocode 3 column strength curves under nonconservative forces is proposed based on a nonlinear eigenvalue analysis algorithm.
4. To illustrate the accuracy and the effectiveness of the proposed method from numerical examples, buckling loads or K -factors of steel columns in frame structures subjected to the conservative or nonconservative forces are evaluated and compared with those by available references.
5. Especially, emphasis is given in showing the effects of the nonconservativeness parameter, the stiffness and span ratios of beam to column on buckling loads. K -factors are also investigated for the inelastic nonconservative systems.

2. FINITE ELEMENT MODEL FOR NONCONSERVATIVE SYSTEM

Figure 1 shows the cantilevered column of the length L subjected to the combined conservative force P_c and the tangential follower force P_t acting at the tip. The former may be thought of as a dead load, while the latter can be considered as a typical follower force and the two forces P_c and P_t are assumed to be constant along the elastic line. For a small deflection, the angle between the x -axis and the tangent to the elastic line of column can be approximated by $\partial v/\partial x$ where v is the lateral displacement. The direction of the resultant subtangential force P is specified by $\alpha \partial v(L)/\partial x$. Thus the parameter α can also be referred to as the nonconservativeness parameter and one can easily derive the following relation.

$$\mathbf{P} = \mathbf{P}_c + \mathbf{P}_t, \quad \alpha = \frac{P_t}{P} \quad (1)$$

Note that the force is conservative for $\alpha=0$, while it is nonconservative for $\alpha \neq 0$ and is a purely tangential follower force for $\alpha=1$.

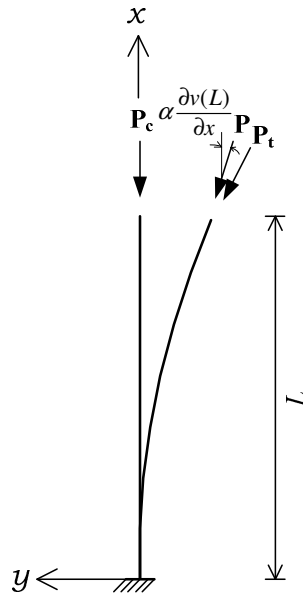


Figure 1. Column subjected to the Nonconservative Force

In order to present the finite element formulation of the element with the length l_e , three nodal displacements at nodal end points p and q ($u^p, v^p, \theta^p; u^q, v^q, \theta^q$) are considered in which u , v and θ are the axial displacement, the lateral displacement, and the angle of rotation, respectively. In this study, the strain energy Π_E of the system, the potential energy Π_G including the work done by the conservative component of the subtangential force and the virtual work $\delta\Pi_{NC}$ by the nonconservative component of the acting force are considered. Here the type of instability is only divergence and the critical flutter load from the dynamic criterion is not considered. These terms are expressed as follows:

$$\Pi_E = \sum_e \frac{1}{2} \int_0^{l_e} \left\{ E_t A \left(\frac{\partial u}{\partial x} \right)^2 + E_t I \left(\frac{\partial^2 v}{\partial x^2} \right)^2 \right\} dx \quad (2a)$$

$$\Pi_G = -\sum_e \frac{1}{2} \int_0^{l_e} P \left(\frac{\partial v}{\partial x} \right)^2 dx \quad (2b)$$

$$\delta \Pi_{NC} = -\alpha P \frac{\partial v(L)}{\partial x} \delta v(L) \quad (2c)$$

where E_t is the tangent modulus; A and I are the cross-sectional area and the second moment of inertia, respectively; δ is the variation. The extended Hamilton principle of the column can be presented by

$$\int_{t_1}^{t_2} [\delta(\Pi_E + \Pi_G) - \delta \Pi_{NC}] dt = 0 \quad (3)$$

The present column theory is implemented via a displacement-based finite element method. For this, the generalized displacements are expressed over each element as an one-dimensional Lagrange interpolation function for the axial displacement u and the third-order Hermitian interpolation function for the lateral displacement v . By substituting the interpolated displacements into Eq. 3, the resulting equation for a single element is obtained in a matrix form as

$$\int_{t_1}^{t_2} [\delta \mathbf{U}_e (\mathbf{K}_e + \mathbf{K}_g - \mathbf{K}_{nc}) \mathbf{U}_e - \delta \mathbf{U}_e \mathbf{F}_e] dt = 0 \quad (4)$$

where \mathbf{U}_e and \mathbf{F}_e are the nodal displacement and force vectors, respectively; \mathbf{K}_e and \mathbf{K}_g are the elastic stiffness matrix and the geometric stiffness matrix due to conservative compressive forces, respectively; \mathbf{K}_{nc} is the load correction stiffness matrix due to the directional change of nonconservative forces.

Finally, the equilibrium equation for whole column members is expressed in a matrix form using the direct stiffness method as follows:

$$(\mathbf{K}_E + \mathbf{K}_G - \mathbf{K}_{NC}) \mathbf{U} = \mathbf{F} \quad (5)$$

where \mathbf{K}_E , \mathbf{K}_G , and \mathbf{K}_{NC} are respectively the elastic and geometric stiffness matrices, and the load correction stiffness matrix due to the nonconservative forces in the global coordinate system.

3. TRADITIONAL ELASTIC AND INELASTIC STABILITY APPROACHES

A wide range of methods have been suggested in the engineering literature for the design calculation of column buckling loads and their corresponding K -factors. This section outlines the broad categories of the traditional elastic and inelastic stability approaches based on the physical extent of the structure under the conservative forces.

3.1 Isolated Subassembly Approach

The effective length of an end-restrained column is defined as the length of an equivalent pinned-ended column that will give the same critical load as the end-restrained column. This effective length can be visualized as the distance between the two inflection points (real and imaginary) of the buckled shape of the end-restrained member. The most widely used method for evaluating K -factor is probably the isolated subassembly approach, also commonly referred to as the alignment chart method or the G -factor method since it offers a straightforward method of obtaining K of the column.

The transcendental equations of the buckling solution of the subassembly for the sideway inhibited and uninhibited frames subjected to conservative compressive forces are given in Eq. 6 and the detailed step-by-step derivation of equations governing the buckling of this assembly is presented by Chen and Lui [32] based on the slope deflection equations.

$$\frac{G_A G_B}{4} \left(\frac{\pi}{K} \right)^2 + \left(\frac{G_A + G_B}{2} \right) \left(1 - \frac{\pi/K}{\tan(\pi/K)} \right) + \frac{2 \tan(\pi/2K)}{\pi/K} - 1 = 0 \quad (6a)$$

$$\frac{G_A G_B (\pi/K)^2 - 36}{6(G_A + G_B)} - \frac{\pi/K}{\tan(\pi/K)} = 0 \quad (6b)$$

where G_A and G_B are the stiffness ratios (G -factors) of columns and beams at joints A and B , respectively, and the detailed expressions are as follows:

$$G_A = \frac{\sum_A (E_c I_c / L_c)}{\sum_A (E_b I_b / L_b)} \quad \text{and} \quad G_B = \frac{\sum_B (E_c I_c / L_c)}{\sum_B (E_b I_b / L_b)} \quad (7a,b)$$

where subscript c and b denote the column and the beam, respectively. Note that Eq. 6 may be analytically derived under several fundamental assumptions that the structure should follow in order to guarantee a reasonable result.

It is well known that the significant economy may be gained by utilizing inelastic buckling solutions in many design situations [13,15], although the column strength calculations are often based on the elastic models. This is because the slenderness of the columns in typical steel buildings is usually small enough such that the distributed yielding occurs along the column lengths before the design strength of the most critically-loaded components is reached. To reflect the inelastic effects into the design of steel frames, the elastic modulus can be lowered with the inelastic stiffness reduction factor τ_a as follows (AISC [2]):

$$E_t = \tau_a E \quad (8)$$

where

$$\tau_a = 1.0 \quad \text{for} \quad P_n / P_y \leq 0.39 \quad (9a)$$

$$\tau_a = -2.724(P_n / P_y) \ln(P_n / P_y) \quad \text{for} \quad P_n / P_y > 0.39 \quad (9b)$$

where E is the elastic modulus; P_n and P_y are the nominal column strength and the yielding axial force, respectively. Thus the inelastic K -factor can be determined by using τ_a times E_c for E_c in the equations for G_A and G_B in Eq. 7

3.2 Story-based Approach

Since the isolated subassembly approach assumes that the shear force of the column does not transfer to other columns in the same story, each of columns in any story buckles in the sideway mode, independent of the adjacent columns. This assumption may be inadequate in some real cases, such as a structural system with leaning columns. To account for the interaction effect between the columns

in the story, the inelastic K -factor can be inferred from the elastic K -factor which the commentary of AISC [2] recommends based on the story buckling method as follows:

$$K = \sqrt{\frac{\pi^2 \tau_a EI / L^2}{P_r} \left(\frac{\sum P_r}{\sum \frac{\pi^2 \tau_a EI}{(K_{n2} L)^2}} \right)} \geq \sqrt{\frac{5}{8}} K_{n2} \quad (10)$$

where K_{n2} is defined as the K value determined directly from the alignment chart, P_r is the required axial compressive force for the rigid column, and $\sum P_r$ is the total axial compressive force of columns in the story. The K -factor obtained from Eq. 10 inherently account for the inelastic effects and the interactions within and between stories at incipient buckling of the system subjected to conservative forces.

On the other hand, the story stiffness method considers later displacements of the story instead of the sum of the compressive force. The inelastic K -factor can be deduced from the Equation C-C2-5 in AISC [2] as follows:

$$K = \sqrt{\frac{\sum P_r}{(0.85 + 0.15R_L)P_r} \left(\frac{\pi^2 \tau_a EI}{L^2} \right) \left(\frac{\Delta_H}{\sum HL} \right)} \geq \sqrt{\frac{\pi^2 \tau_a EI}{L^2} \left(\frac{\Delta_H}{1.7HL} \right)} \quad (11)$$

where H and Δ_H are respectively the shear in the column and the lateral displacement of the story under consideration. R_L is the ratio of the vertical column load for all leaning columns in the story to the vertical load of all the columns in the story expressed by

$$R_L = \frac{\sum P_r \text{ leaning columns}}{\sum P_r \text{ all columns}} \quad (12)$$

4. SYSTEM BUCKLING APPROACH UNDER NONCONSERVATIVE FORCES

In this section, the elastic and inelastic system buckling approaches for calculating buckling loads and K -factors for columns in steel frames subjected to nonconservative forces are presented.

4.1 Column Strength Curves for AISC-LRFD and Eurocode 3

The inelastic buckling of columns can lead to system collapse prior to the formation of a plastic hinge on beam members as external loads are increased on the steel frames. In several design specifications, the load-carrying capacity of the column member is modified based on such information as theoretical buckling analysis results, various experimental results come from initial imperfections due to manufacturing or constructional errors, residual stress due to welding, dependency on cross-sectional shape and etc.

In this study, two kinds of load-carrying capacity curves are presented for further development in inelastic buckling formulation, one from the AISC-LRFD [33] and the other from the Eurocode 3 [7]. In the development of the LRFD specification, the AISC specification uses a column strength curve as

$$f^* = 0.658\lambda^{*2} \quad \text{for } 0 \leq \lambda^* \leq 1.5 \tag{13a}$$

$$f^* = \frac{0.877}{\lambda^{*2}} \quad \text{for } \lambda^* > 1.5 \tag{13b}$$

where

$$f^* = \frac{f_{cr}}{f_y}, \quad \lambda^* = \frac{KL}{\pi r} \sqrt{\frac{f_y}{E}} \tag{14a,b}$$

In which f^* and λ^* are the dimensionless strength of the column and the slenderness parameter, respectively; f_{cr} and f_y are the critical buckling stress and the specified minimum yield stress, respectively; r the governing radius of gyration about the axis of buckling. This equation was calibrated to fit closely the SSRC curve 2 [34], which was modified to reflect an initial out-of-straightness of about 1/1500. This strength curve of AISC-LRFD [33] is depicted in Figure 2 together with the Euler hyperbola curve and the strength curves of Eurocode 3 [7].

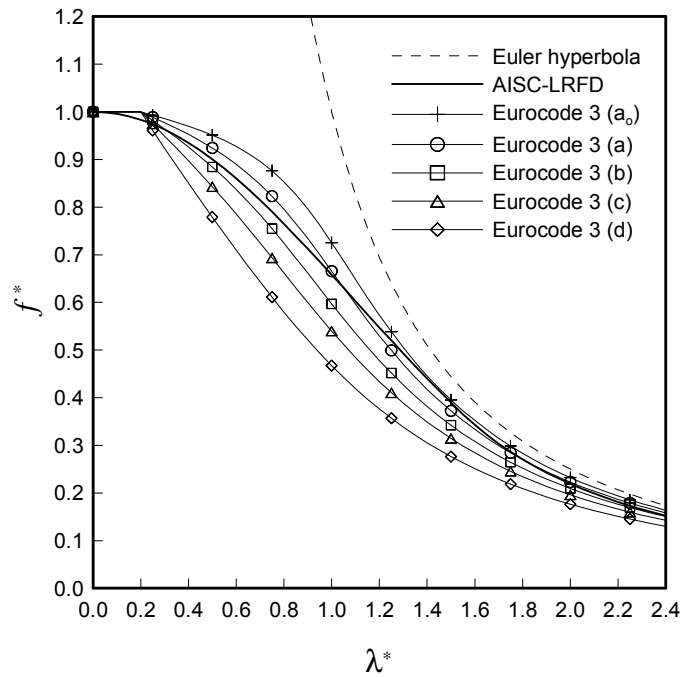


Figure 2. Load-carrying Capacity Curves in AISC-LRFD and Eurocode 3

The simplified design predictions for the maximum load (or resistance) were also used in Eurocode 3 [7] that had been developed from the results of computer analyses and correlations with available test data considering the residual stresses and the initial imperfection in design of compression members. The Eurocode 3 approach to determine the buckling resistance of compression member is based on the same principles as that of BS 5950 [35]. Although minor technical differences exist, the primary difference between the two codes is in the presentation of the method. The basic formulations for buckling curves are given by

$$f^* = \frac{1}{\phi + \sqrt{\phi^2 - \lambda^{*2}}} \quad \text{but } f^* \leq 1.0 \tag{15}$$

where

$$\phi = 0.5 \left[1 + \beta (\lambda^* - 0.2) + \lambda^{*2} \right] \tag{16}$$

In which β is the imperfection factor which shifts the resistance curve, as shown in Figure 2, for different cross-section types, properties, thicknesses, buckling axes, and material strengths and the five values of β for each of these curves are given in Table 1. The advantage of this approach is that the buckling resistances of a particular group of sections can be determined by assigning an appropriate value of β to them.

Table 1. Imperfection Factors for Buckling Curves

Buckling curve	a_o	a	b	c	d
Imperfection factor β	0.13	0.21	0.34	0.49	0.76

4.2 Elastic and Inelastic System Buckling Analyses

According to the study by Chen and Lui [32], the best way to evaluate the more accurate K -factor for each individual member of the frame structure should be the stability analysis of the entire structure as a whole. The followings describe the elastic and inelastic system buckling analysis procedure proposed by this study.

Step 1) Constitute the system elastic stiffness matrix \mathbf{K}_E for the whole structure. For a given load condition, the following linear elastic analysis is performed from Eq. 5 to evaluate the axial force P_j for each member where subscript j denotes the number of member.

$$\mathbf{K}_E \mathbf{U} = \mathbf{F} \tag{17}$$

Step 2) Solve the linear eigenvalue problem in Eq. 18 and calculate the minimum eigenvalue ξ_{cr}^i where superscript i means the number of iteration.

$$\mathbf{K}_E^i (E_t^i) \Delta \mathbf{U} = \xi_{cr}^i (\mathbf{K}_G^i - \mathbf{K}_{NC}^i) \Delta \mathbf{U} \tag{18}$$

Then the elastic buckling load of each member is calculated as $\xi_{cr}^i P_j$.

Step 3) Evaluate the slenderness parameter λ_j^{*i} of each member as follows:

$$\lambda_j^{*i} = \frac{L_e^i}{\pi r} \sqrt{\frac{f_y}{E}} \tag{19}$$

where the effective length is defined as

$$L_e^i = K^i L = \pi \sqrt{\frac{E_{t,j}^i I_j}{\xi_{cr}^i P_j}} \tag{20}$$

Step 4) Calculate the inelastic critical buckling stress $f_{cr,j}^i$ of each member from the AISC-LRFD and Eurocode 3 column strength curves by utilizing the slenderness parameter λ_j^{*i} .

Step 5) Calculate the refined tangent modulus of each member as

$$E_{t,j}^{i+1} = \frac{f_{cr,j}^i}{f_{cr,in,j}^i} E_{t,j}^i \tag{21}$$

where $f_{cr,in,j}^i$ is the inelastic buckling stress of each member obtained from Eq. 18. Next check the convergence designated as $(E_{t,j}^{i+1} - E_{t,j}^i) / E_{t,j}^i < \epsilon$. In case that the convergence is not achieved, go back to the step 2).

Step 6) Determine the final K -factor of each member.

In addition, Figure 3 shows the schematic procedures for the evaluation of K -factor in inelastic stability analysis.

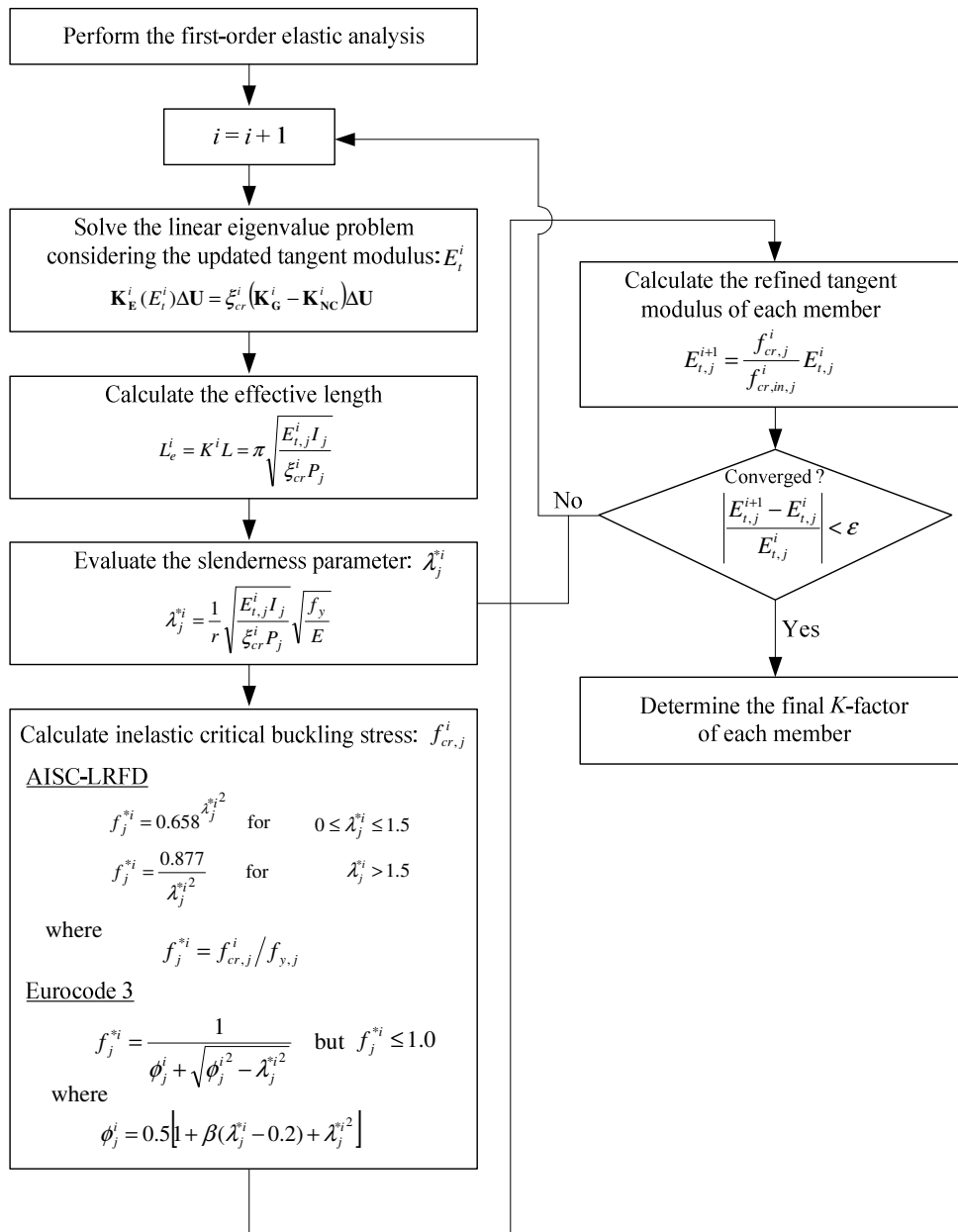


Figure 3. Flowchart of the Evaluation of K -factor in Inelastic Stability Analysis

5. NUMERICAL EXAMPLES

In numerical examples, to show the accuracy and the usefulness of the present study, the elastic and inelastic stability analyses of the frame structures subjected to conservative or nonconservative forces are performed and compared with other available references. The effects of various parameters on buckling loads and K -factors for columns in steel frames are investigated. In subsequent examples, each column is modeled with the five finite beam elements and the yield stress f_y of all columns is 344.7 MPa (50 ksi).

5.1 Three-story Frame Subjected to Conservative Forces

The stability analysis is carried out on the three-story frame subjected to conservative forces to illustrate the validity of the present method to predict K -factors of columns. Figure 4 shows the configuration of the frame with W-shape section, and its material and sectional properties are as follows: $E = 2 \times 10^5$ MPa; W8×35: $A = 6.645 \times 10^{-3} \text{ m}^2$, $I = 5.286 \times 10^{-5} \text{ m}^4$; W8×48: $A = 9.097 \times 10^{-3} \text{ m}^2$, $I = 7.659 \times 10^{-5} \text{ m}^4$; W14×30: $A = 5.71 \times 10^{-3} \text{ m}^2$, $I = 1.2112 \times 10^{-4} \text{ m}^4$; W21×44: $A = 8.387 \times 10^{-3} \text{ m}^2$, $I = 3.5088 \times 10^{-4} \text{ m}^4$. The K -factors evaluated by present elastic and inelastic stability analyses are presented in Table 2. For comparison, the K -factors by Liu and Xu [17] who uses the geometrical stiffness distribution approach for elastic analysis and AISC [2] using the modified equation for determining K -factors based on the story buckling approach and the alignment chart method for elastic and inelastic analyses are presented. It can be seen that the results by the present Euler hyperbola curve show fairly good agreement with those by Liu and Xu [17]. When the inelastic behavior is considered, the K -factor for column C_1 , on which the highest axial force acts, becomes smaller. On the other hand, the inelastic system buckling analysis introduces the higher K -factor for column C_3 having the relatively small axial force.

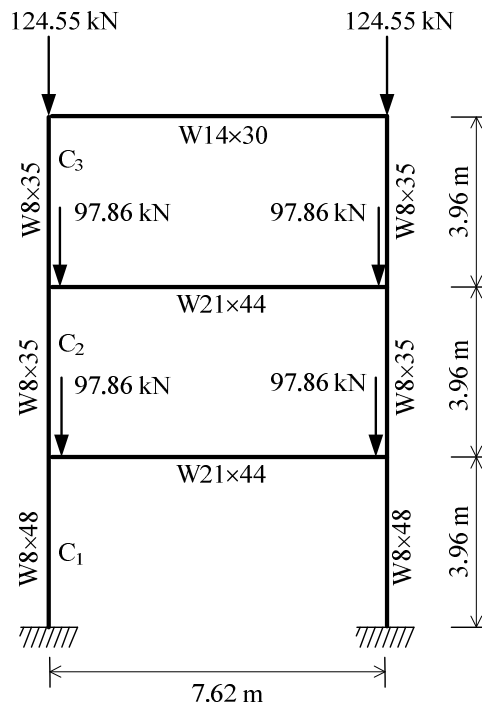


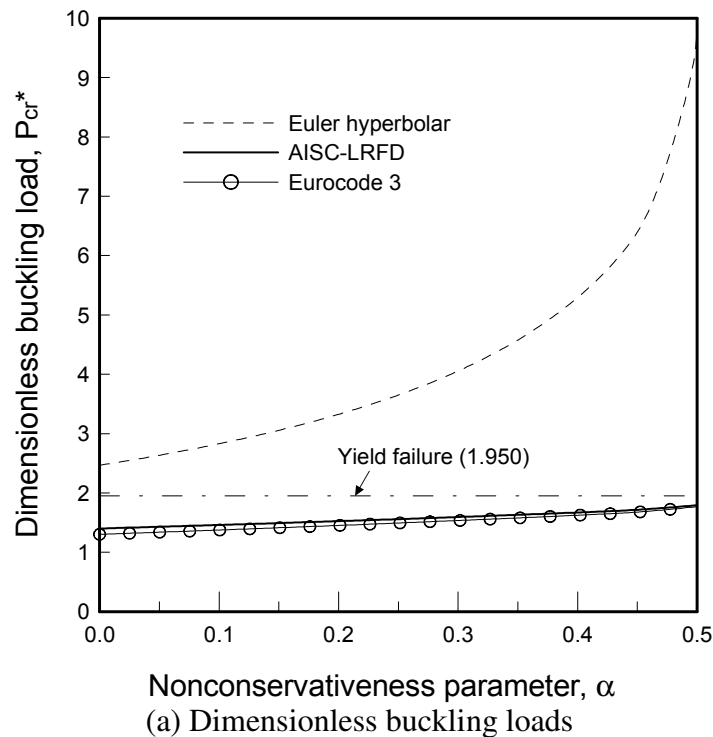
Figure 4. Three-story Frame subjected to Conservative Forces

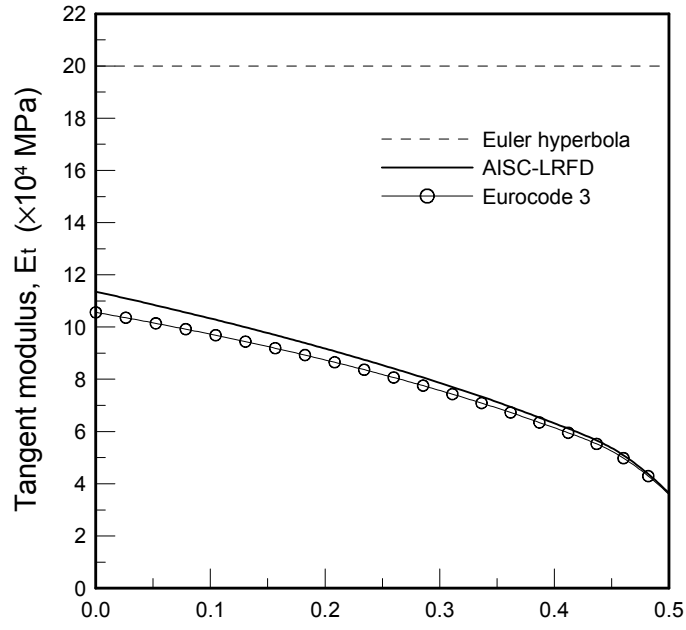
Table 2. K -factors of the Three-story Frame Subjected to Conservative Forces ($\alpha = 0$)

Methods	Elastic analysis			Inelastic analysis			
	C_1	C_2	C_3	C_1	C_2	C_3	
Liu and Xu [17]	1.15	1.15	1.53	-	-	-	
Story buckling approach [2]	1.11	1.21	1.23	1.07	1.14	1.21	
Alignment chart method	1.11	1.21	1.23	1.07	1.14	1.21	
This study	Euler hyperbola	1.15	1.14	1.53	-	-	-
	AISC-LRFD	-	-	-	1.03	1.11	2.31
	Eurocode 3	-	-	-	1.03	1.10	2.13

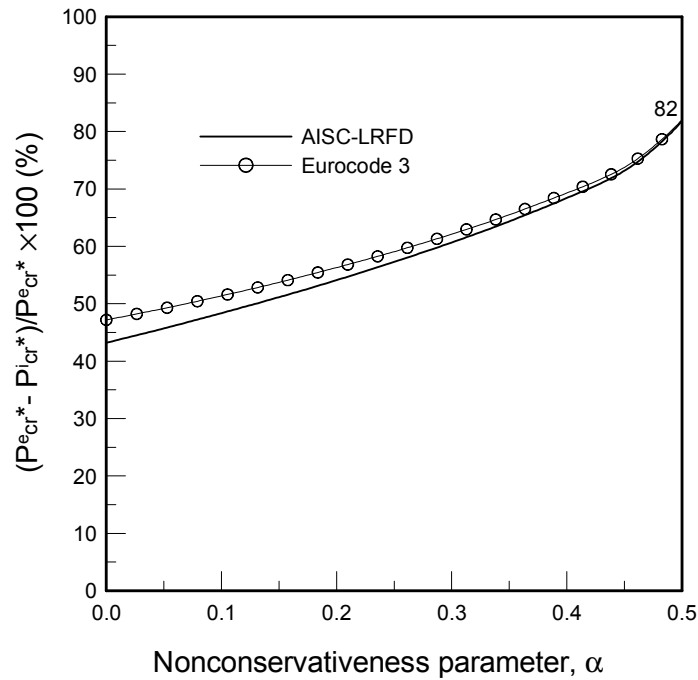
5.2 Cantilevered Column

For verification purpose of the present study in evaluation of buckling load for the column subjected to the nonconservative force, we consider the cantilevered column with W-shape section (W8×35) subjected to the compressive tangential force P acting at the top of the column as shown in Figure 1. The length of column is 3 m and the material and sectional properties are as follows: $E = 2 \times 10^5$ MPa, $A = 6.645 \times 10^{-3}$ m², $I = 5.286 \times 10^{-5}$ m⁴. For simplicity the dimensionless buckling load $P_{cr}^* = P_{cr} L^2 / (EI)$, in which P_{cr} denotes the critical buckling load, is introduced. In Table 3, the dimensionless buckling loads by this study are compared with results obtained by Rao and Rao [36] deriving the exact solutions, with the solutions given by Leipholz [37], and with those proposed by Kikuchi [38] via a finite element technique. It can be found from Table 3 that the results by this Euler hyperbola curve are in excellent agreement with other literature results for whole α considered.





(b) Variation of the tangent modulus



(c) Effects of inelasticity on the dimensionless buckling loads

Figure 5. Dimensionless Buckling Loads and Its Corresponding Tangent Modulus, and the Effects of Inelasticity on the Dimensionless Buckling Loads for the Cantilevered Column with Respect to α

Table 3. Dimensionless Buckling Loads for the Cantilevered Column Subjected to the Nonconservative Force

Methods	Nonconservativeness parameter α						
	0.0	0.1	0.2	0.3	0.4	0.5	
Rao and Rao [36]	2.467	2.830	3.325	4.055	5.292	9.870	
Leipholz [37]	2.467	2.830	3.325	4.055	5.292	9.870	
Kikuchi [38]	2.467	2.830	3.325	4.055	5.293	9.872	
This study	Euler hyperbola	2.467	2.830	3.325	4.055	5.293	9.871
	AISC-LRFD	1.401	1.461	1.526	1.595	1.671	1.795
	Eurocode 3	1.303	1.376	1.453	1.536	1.625	1.771

Figure 5(a) shows the variation of the dimensionless buckling loads with respect to α . It can be observed from Figure 5(a) that the buckling loads from both elastic and inelastic stability analyses increase as α increases, so the conservative case ($\alpha = 0$) is the more susceptible of instability. Also the increase ratio of buckling load with respect to α for the Euler hyperbola curve is larger than those for AISC-LRFD and Eurocode 3 curves. As is expected, this is due to the fact that as α increases, the critical buckling loads from the inelastic stability analyses increases slowly with significant degradation of the tangent modulus as seen in Figure 5(b). In Figure 5(c), the inelastic effects on the dimensionless buckling load are plotted with α . Here P_{cr}^{e*} and P_{cr}^{i*} denote the dimensionless buckling loads from the elastic and inelastic stability analyses, respectively. As the results show, the inelastic effects on the buckling loads increase as α increases up to around 82% at $\alpha = 0.5$ for both of AISC-LRFD and Eurocode 3 curves.

5.3 Frame with Semirigid Connections

The frame with semirigid connections as shown in Figure 6 is considered. The frame consists of columns with W14×22 section and the modulus of elasticity is 2×10^5 MPa. The moment of inertia for columns (I_c) and beams (I_{b1}, I_{b2}) are as follows: $I_c = 82.83 \times 10^{-6} \text{ m}^4$, $I_{b1} = 83.25 \times 10^{-6} \text{ m}^4$, and $I_{b2} = 10.41 \times 10^{-6} \text{ m}^4$.

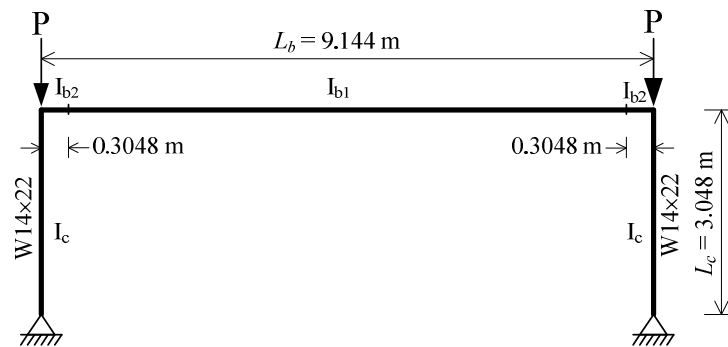


Figure 6. Frame with Semirigid Connections

When the columns are subjected to conservative forces ($\alpha = 0$), the present K -factors are compared with one obtained by Cheong-Siat-Moy [39] in Table 4. Cheong-Siat-Moy [39] proposed a new design method employing a fictitious lateral load that was a fraction of the gravity loads. As can be seen in Table 4, the result from this Euler hyperbola curve is in good agreement with that from Cheong-Siat-Moy [39]. It can also be found that the inelastic effects decrease the K -factor by 16.6% and 15.6% for AISC-LRFD and Eurocode 3 curves, respectively.

Table 4. *K*-factors for the Column of Frame with Semirigid Connections Subjected to Conservative Forces

Cheong-Siat-Moy [39]	This study		
	Euler hyperbola	AISC-LRFD	Eurocode 3
3.8	3.85	3.21	3.25

Table 5 summarizes buckling loads and *K*-factors for columns with respect to α . It is seen that the buckling load increases by tangential follower forces ($\alpha = 1.0$) by 269.8% for the elastic analysis. On the other hand, for the inelastic columns, the buckling loads increase by 23.1% and 25.6% for AISC-LRFD and Eurocode 3 curves, respectively, due to the tangential forces. In case of *K*-factors, as α increases, the *K*-factors converge to the value of 2.0 for both of the elastic and inelastic analyses. This *K*-factor of 2.0 corresponds to one in column with clamped boundary condition at the base, and the rotation fixed and translation free one at upper end. It is also seen that the tangential forces decrease *K*-factor by 48.1% for Euler hyperbola and 37.7% and 38.5% for AISC-LRFD and Eurocode 3 curves, respectively.

Table 5. Buckling Loads and *K*-factors for the Column of Frame with Semirigid Connections Subjected to Nonconservative Forces

Methods	Nonconservativeness parameter α						
	0.0	0.2	0.4	0.6	0.8	1.0	
Buckling loads	Euler hyperbola	1189.8	1402.0	1704.0	2164.9	2938.9	4399.7
	AISC-LRFD	1021.7	1092.7	1158.0	1200.4	1235.3	1258.2
	Eurocode 3	1034.5	1125.8	1198.3	1245.0	1279.1	1299.1
<i>K</i> -factors	Euler hyperbola	3.85	3.54	3.21	2.85	2.45	2.00
	AISC-LRFD	3.21	2.87	2.58	2.34	2.15	2.00
	Eurocode 3	3.25	2.92	2.61	2.35	2.15	2.00

Figure 7 shows buckling loads and inelastic effects on buckling loads with respect to α . It is observed from Figure 7(a) that the buckling loads increase with increase of α and the elastic buckling load is smaller than the yield failure load at $0 \leq \alpha \leq 0.24$ and it is larger at $\alpha > 0.24$. In Figure 7(b), P_{cr}^e and P_{cr}^i mean the buckling loads obtained from the elastic and inelastic analyses, respectively. The effects of inelasticity on buckling loads increase for increasing values of α by 71.4% and 70.5% for AISC-LRFD and Eurocode 3, respectively. Figure 8 presents the variation of *K*-factors and the effects of inelasticity on *K*-factors. The *K*-factors decrease with increase of α , as shown in Figure 8(a). In Figure 8(b), K^e and K^i are the *K*-factors evaluated from the elastic and inelastic analyses, respectively. It is observed from Figure 8(b), the effects of inelasticity increase with increase of α and have the maximum values of 19.6% and 18.7% for AISC-LRFD and Eurocode 3 curves, respectively, at $\alpha = 0.4$. These effects decrease thereafter and become to be zero at $\alpha = 1.0$.

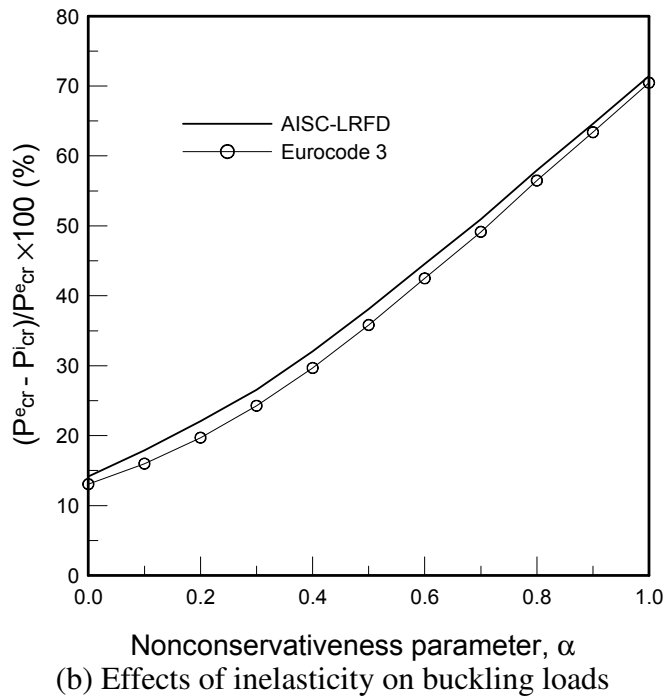
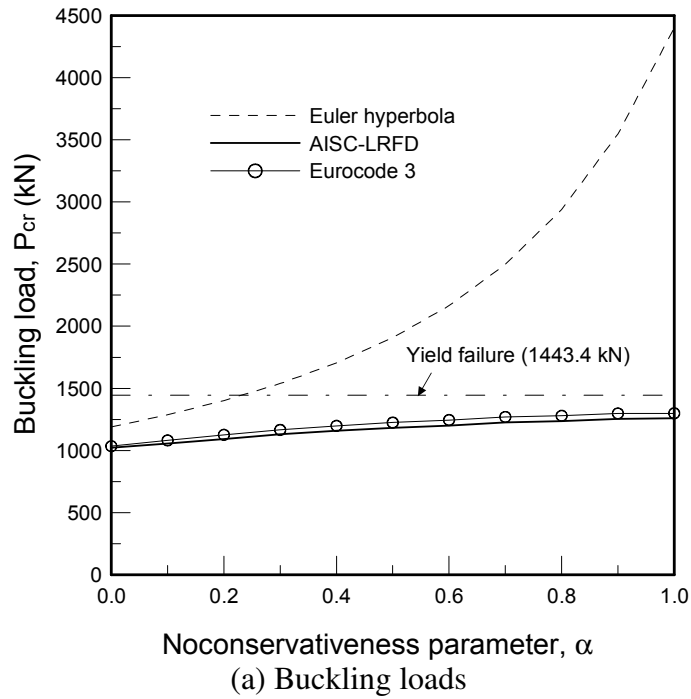
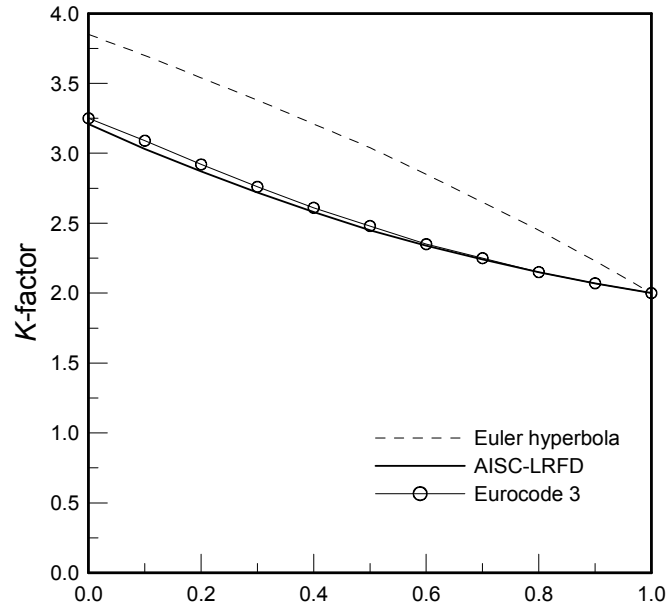
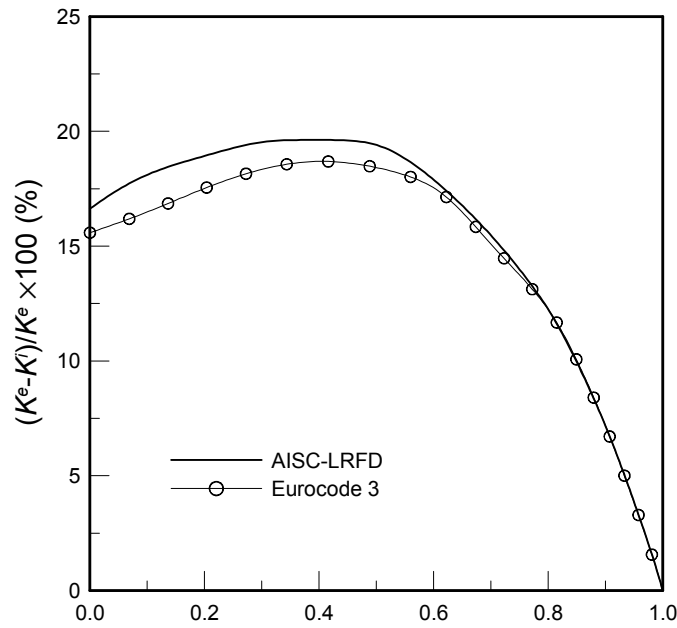


Figure 7. Buckling Loads and the Effects of Inelasticity on Buckling Loads for the Frame with Semirigid Connections with Respect to α



(a) *K*-factors

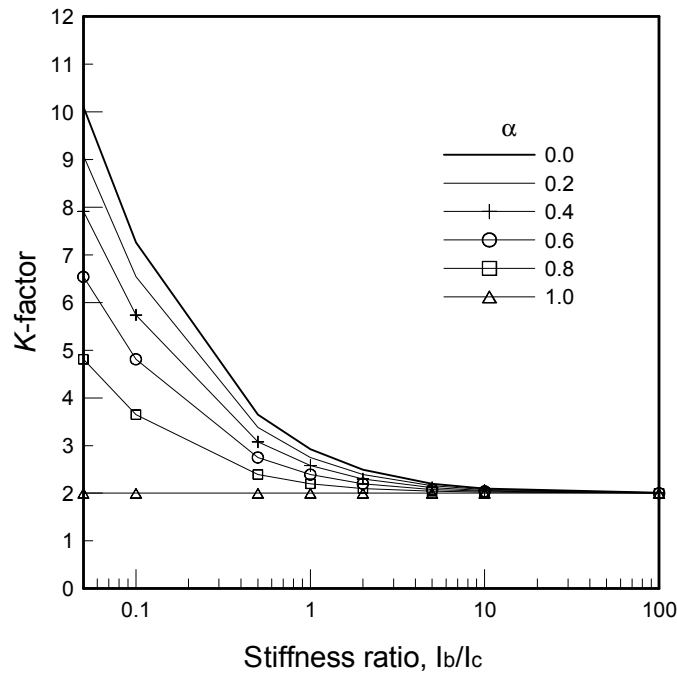


(b) Effects of inelasticity on *K*-factors

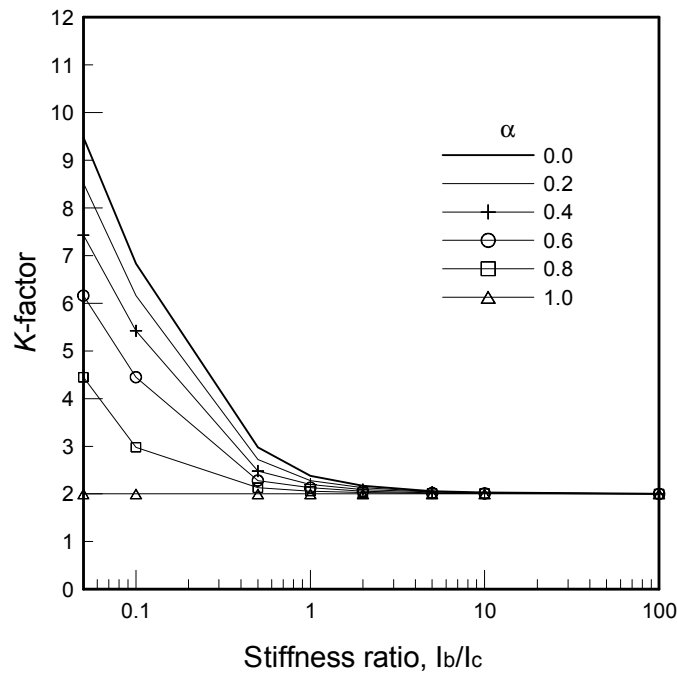
Figure 8. *K*-factors and the Effects of Inelasticity on *K*-factors for the Frame with Semirigid Connections with Respect to α

Now for the frame with rigid connections ($I_{b2} = I_{b1} = 83.25 \times 10^{-6} \text{ m}^4$) subjected to nonconservative forces, the variation of *K*-factors and the effects of inelasticity on *K*-factors are investigated with respect to various values of the stiffness ratio I_b/I_c and the span ratio L_b/L_c of beam to column. First, Figures 9(a) and 9(b) show the variation of *K*-factors with values of α versus the stiffness ratio for the Euler hyperbola and AISC-LRFD curves, respectively. In this example, we keep the moment of inertia of column I_c constant and change the value of I_b . It is observed that *K*-factors from the present elastic and inelastic stability analyses decrease as I_b/I_c increases and converge to the value

of 2.0 after I_b/I_c of 10. This means that if the stiffness of the beam approaches infinity, the columns will behave like a member with a hinged boundary condition at the base, and a rotation fixed and translation free one at the upper end. The effects of inelasticity on K -factors are plotted in Figure 9(c) with respect to I_b/I_c where K^A is the K -factor evaluated from AISC-LRFD curve. It can also be seen that the K -factors from the inelastic analysis is smaller than those from elastic one except for $\alpha = 1.0$. It is interesting to observe from Figure 9(c) that the maximum values of inelastic effects are about 19.8% for whole α considered except for $\alpha = 1.0$. The values of I_b/I_c corresponding to the largest effects of inelasticity are on the decrease as α increases. This is due to the fact that as α increases, the total stiffness decreases, as shown in Eq. 5, and therefore the maximum inelastic effect occurs at the stiffness ratio with more flexible beam stiffness.



(a) K -factors from the Euler hyperbola curve



(b) K -factors from the AISC-LRFD curve

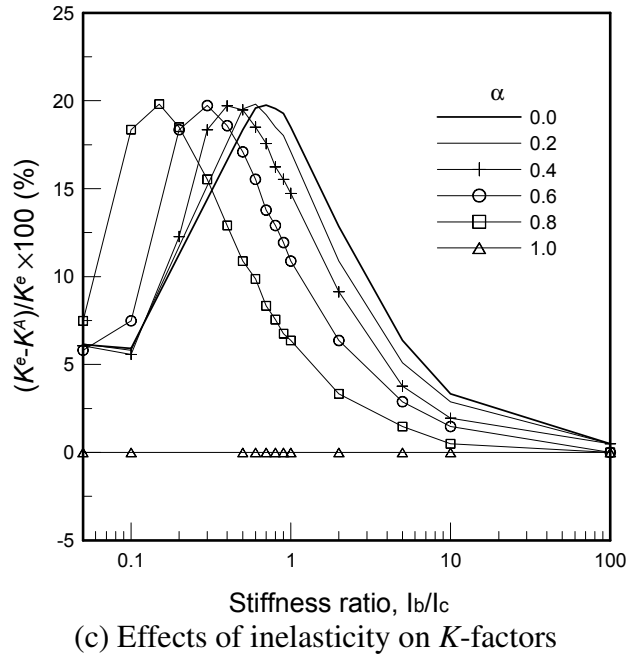
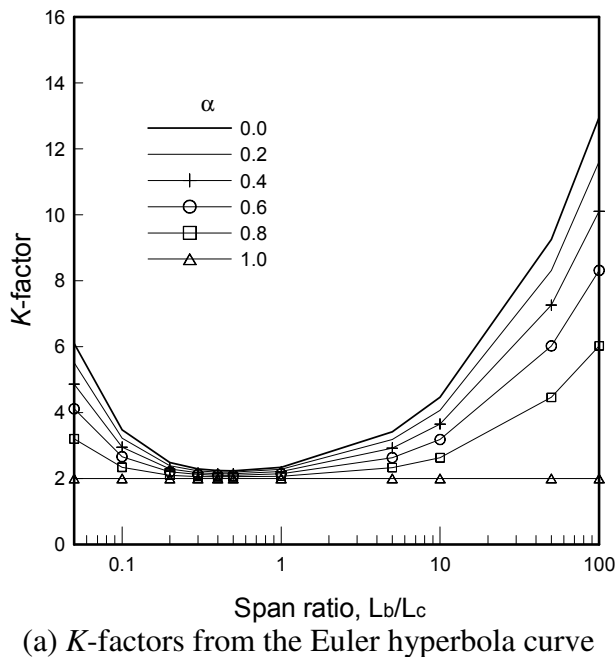
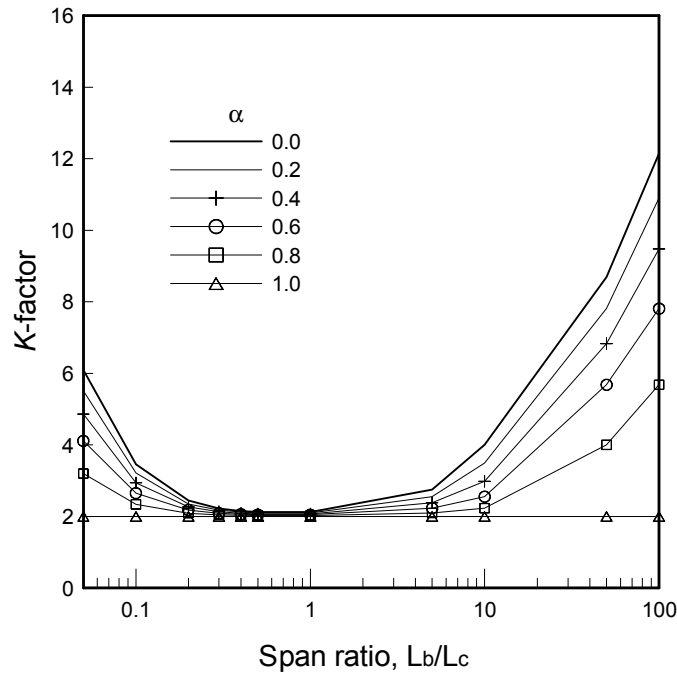


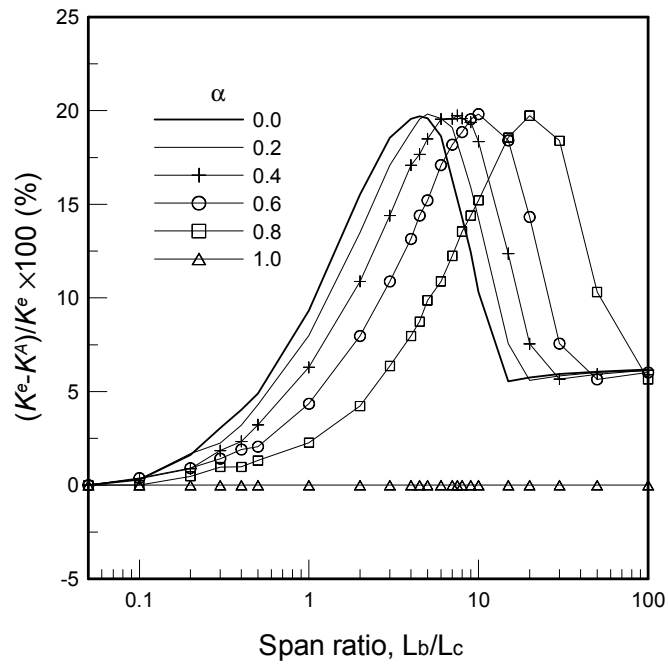
Figure 9. K -factors and the Effects of Inelasticity on K -factors for the Frame with Rigid Connections with Respect to I_b/I_c

Next, the variation of K -factors from the Euler hyperbola and AISC-LRFD curves are presented in Figures 10(a) and 10(b), respectively, with respect to the span ratio L_b/L_c as the length of column remains constant and that of beam is varied. As can be seen, the K -factors for the frame subjected to tangential forces ($\alpha = 1.0$) are irrespective of L_b/L_c . It is interesting to observe that as L_b/L_c increases, the K -factors decrease and have the minimum values at L_b/L_c of 0.5 and increase thereafter for both of the Euler hyperbola and AISC-LRFD curves. As shown in Figure 10(c), the values of inelastic effects on K -factors increase with increase of L_b/L_c and have the maximum value of 19.7% for whole α except $\alpha = 1.0$, and decrease thereafter. The values of L_b/L_c corresponding to the largest effects of inelasticity are on the increase as α increases.





(b) K -factors from the AISC-LRFD curve



(c) Effects of inelasticity on K -factors

Figure 10. K -factors and the Effects of Inelasticity on K -factors for the Frame with Rigid Connections with Respect to L_b/L_c

6. CONCLUSIONS

The present paper proposes the inelastic system buckling analysis method to determine buckling loads and K -factors for columns of steel frames subjected to nonconservative forces by generalizing the iterative buckling procedures. This study overcomes the limit of traditional elastic and inelastic stability analyses using the many unrealistic assumptions which may lead to unreasonable K -factor if the frame structure does not satisfy these assumptions. Moreover this study is the first attempt to perform the stability analysis of the frame structures subjected to nonconservative forces considering column strength curve which considers the geometric imperfections and the residual stresses of the compression members. Throughout numerical examples, comparative examples and parametric studies are conducted and the following conclusions are drawn.

- 1) For evaluation of buckling loads and K -factors for the elastic conservative and nonconservative systems, the results by this study are in good agreement with those from other researchers.
- 2) For column subjected to the smaller axial force of in frame, the K -factor from inelastic analysis is larger than that from elastic analysis but it is the reverse for column subjected to the larger axial force.
- 3) For both elastic and inelastic analyses, the critical buckling loads of the frame increase with increase of α , and so the conservative case ($\alpha = 0$) is the more susceptible of instability.
- 4) The increase ratio of the inelastic buckling load of the frame with respect to α is smaller than that of the elastic buckling load due to the significant degradation of the tangent modulus in inelastic analysis.
- 5) For all α , the K -factors from the elastic and inelastic analyses decrease and converge to the specified value as I_b/I_c increases.
- 6) The maximum values of inelastic effects on K -factors of the frame are same for all α considered except for $\alpha = 1.0$ and the values of I_b/I_c corresponding to the largest effects of inelasticity are on the decrease as α increases.
- 7) As L_b/L_c increases, the K -factors have the minimum values at $L_b/L_c = 0.5$ and increase thereafter for both elastic and inelastic analyses. The values of inelastic effects on K -factors increase with L_b/L_c and have the same maximum values for whole range of α .

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